

Energy Systems – fall 2004 – final exam

This is a CLOSED BOOK, take-home exam, due Friday, 3. Dec. 2004 at 2:00 in the envelope outside the secretary's office, Lab II Rm 2250. You may refer to your homework and notes but no other resources. Do not discuss the exam with anyone or get help from anyone. This is designed as a 2-hour exam, so you should be able to finish it in 3 hours. Do not use calculators or computers for numerical solutions, graphing, or anything else.

Always * show your work clearly, * use words to explain your reasoning, and circle your answer.

(print)

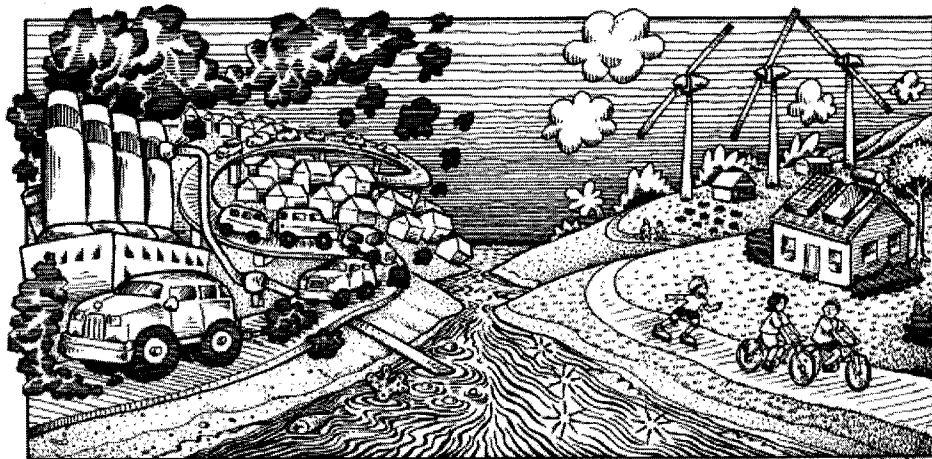
(sign your name)

ZITA - SOLUTIONS

I affirm that I worked this exam without help from anyone, and without using a calculator or computer, texts, or any other resources except my own homework.

Use the best problem-solving techniques you have learned in homework this quarter.

Please do your rough work on scratch paper, and turn in NEAT exam-quality work.



"Global Warming" illustration for Environment Hawaii, August 1999

http://www.pritchettcartoons.com/global_warm.htm

.....Please leave the section below for your prof's grading notes.....

Section A: Physics & Energy, analytical

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Section B: Physics & Energy, numerical

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Section C: precalculus and differentiation

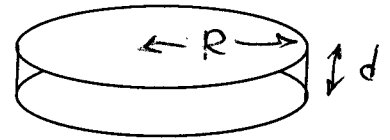
exp, ln, $\frac{d}{dx}$, sin 20

Section D: Go online and do the ESfinalQuiz on inQsit. It will include questions from your classmates' presentations and possibly seminar. It will be available Friday at noon, and is due next Monday at 10 am.

48/48 = 100%

Section A: Find an equation for each quantity, in terms of constants and given variables.

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1. Consider a circular lake with a radius R and a uniform depth d .
 - (a) Label R and d on the diagram.
 - (b) What is the surface area A ? (c) What is the volume V of the lake?
 - (d) If water has density ρ , what is the mass m of water in the lake?



$$A = \pi R^2 \quad V = A \cdot d = \pi R^2 d$$

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \rho = \frac{m}{\pi R^2 d} \rightarrow \boxed{m = \rho \pi R^2 d} = \rho V$$

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2. Say the water in this lake falls down a dam of height h at a rate $k = dm/dt$. [kg/s]
 - (a) What are the units of k ?
 - (b) What is the total energy U this water can produce if it falls from a height h ? $U = mgh$
 - (c) How fast (v) will the water be moving at the bottom of its fall?
 - (d) What is the power P generated by the falling water at any moment?
 - (e) How long (time T) can this lake keep generating energy (at this constant rate, if it is not recharged)?

© $U_{\text{top}} = K_{\text{bottom}}$
 $mgh = \frac{1}{2}mv^2$

$$gh = \frac{1}{2}v^2$$

$$2gh = v^2$$

$$\boxed{v = \sqrt{2gh}}$$

© Power = $\frac{\text{Energy}}{\text{time}} = \frac{mgh}{t} = \frac{dm}{dt}gh = \boxed{kg h = 1}$

© time = $\frac{\text{Energy}}{\text{power}} = \frac{U}{dm/dt}$

$$T = \frac{U}{kg h} = \frac{mgh}{kg h} = \frac{m}{k}$$

3. How much heat Q would it take to evaporate the whole lake, if it starts at temperature T_0 ?

2 $Q = mc\Delta T + mL_v$ where $\Delta T = T_v - T_0$ and $T_v = 100^\circ\text{C} = 373\text{K}$
↑ vaporization

4. If gasoline contains ϵ joules per gallon, (a) how many gallons G of gasoline is equivalent to the gravitational potential energy in this lake?

- 3 (b) How much would that cost C , assuming a price of P dollars/gallon?
 (c) How much would it cost to turn the lake to steam?

© $\epsilon = \frac{\text{energy}}{\text{gallon}} = \frac{U}{G} \rightarrow \boxed{G = \frac{U}{\epsilon}}$

© cost_{gas} = $\frac{\text{dollars}}{\text{gallon}} \times \text{gallons}$

$$\boxed{C = P \cdot G = \frac{PU}{\epsilon}}$$

© cost_{steam} = $\frac{\text{heat energy (J)}}{\text{joule/gallon}} \times \frac{\text{cost}}{\text{gallon}}$

$$\boxed{C_s = \frac{Q}{\epsilon} P}$$

Section B: Calculate solutions to your answers in A, in simplest exact form (e.g. $\sqrt{3\pi} \times 10^7 m$) or order-of-magnitude answer, without using a calculator or computer. Remember to include units. Always start with the equation you generated in Section A.

Data about water: $c = 4190 \text{ J/kg}\cdot\text{K}$, $L_v = 2260 \text{ kJ/kg}$, $L_f = 333 \text{ kJ/kg}$
Let's say the lake is small, just 1 km across, and 10 m deep.

$$R = 0.5 \times 10^3 \text{ m}, \quad d = 10 \text{ m}$$

1(a) Find the density of water in kg/m^3 : $\rho = \frac{1 \text{ kg}}{\text{liter}} \left| \frac{\text{liter}}{10^3 \text{ cm}^3} \right| \left| \frac{10^2 \text{ cm}}{1 \text{ m}} \right|^3 = \frac{10^3 \text{ kg}}{\text{m}^3}$

(b) Surface area of lake = $A = \pi R^2 = \pi \left(\frac{1}{2} \times 10^3 \text{ m} \right)^2 = \frac{\pi}{4} \times 10^6 \text{ m}^2 = A$

(c) Volume of lake = $V = Ad = \frac{\pi}{4} \times 10^6 \text{ m}^2 \times 10 \text{ m} = \frac{\pi}{4} \times 10^7 \text{ m}^3 = V$

(d) Mass of water in lake = $m = \rho V = 10^3 \frac{\text{kg}}{\text{m}^3} \times \frac{\pi}{4} \times 10^7 \text{ m}^3 = \frac{\pi}{4} \times 10^{10} \text{ kg} = m$

2(b) For the rest of this problem, please assume there is $m = 10^{10} \text{ kg}$ of water in the lake and $g = 10 \text{ m/s}^2$. If the water falls 80 m, then find the gravitational potential energy in lake.

$$U = mgh = 10^{10} \text{ kg} \times 10 \frac{\text{m}}{\text{s}^2} \times 80 \text{ m} = 8 \cdot 10^{12} \left(\frac{\text{kg m}^2}{\text{s}^2} = \text{Joules} \right)$$

(c) Speed of water at the bottom = $v = \sqrt{2gh}$

$$= \sqrt{2 \times 10 \frac{\text{m}}{\text{s}^2} \times 80 \text{ m}} = \sqrt{16 \times 10^2 \frac{\text{m}^2}{\text{s}^2}} = 4 \times 10 \frac{\text{m}}{\text{s}} = 40 \frac{\text{m}}{\text{s}}$$

(d) If the water falls at a rate of $k = 30 \text{ kg/s}$, find the power generated by falling water

$$P = kgh = 30 \frac{\text{kg}}{\text{s}} \times 10 \frac{\text{m}}{\text{s}^2} \times 80 \text{ m} = 24 \times 10^3 \frac{\text{kg m}^2}{\text{s}^3} = 2.4 \times 10^4 \text{ Watts}$$

(e) Time that lake generates energy = $T = \frac{m}{k} = \frac{10^{10} \text{ kg}}{30 \text{ kg/s}} = \frac{1}{3} \times 10^9 \text{ s} \approx 3 \times 10^8 \text{ s} \approx 10 \text{ years}$

3. If the lake starts at a temperature of $T_0 = 20^\circ\text{C}$, find the heat required to evaporate the lake. $\Delta T = 100^\circ - 20^\circ\text{C}$

$$Q = m(c\Delta T + L_v) = 10^{10} \text{ kg} \left(4.19 \times 10^3 \frac{\text{J}}{\text{kg}\cdot\text{K}} \times 80 \text{ K} + 2.26 \times 10^6 \frac{\text{J}}{\text{kg}} \right) \quad \Delta T = 80^\circ\text{C} = 80 \text{ K}$$

$$\approx 10^{10} \text{ kg} \left(30 \times 10^3 + 3 \times 10^6 \approx 3.3 \times 10^6 \frac{\text{J}}{\text{kg}} \right) \approx 3.3 \times 10^{16} \text{ J}$$

(Water vapor contains a lot of energy, and can even power hurricanes.)

4. One gallon of gasoline contains $1.3 \times 10^8 \text{ Joules}$. What is $\epsilon = 1.3 \times 10^8 \text{ Joules/gallon}$ ($Q \approx 10^4 \text{ U}$, heat > grav)

(a) Lake's gravitational energy equivalent in gallons of gasoline = $G =$

$$G = \frac{U}{\epsilon} = \frac{8 \times 10^{12} \text{ J}}{1.3 \times 10^8 \text{ J/gal}} \approx 6 \times 10^4 \text{ gal}$$

(b) Assume gas price $P = \$2/\text{gallon}$. Find the cost for this much gasoline = $C =$

$$C = P \cdot G = \frac{\$2}{\text{gal}} \times 6 \times 10^4 \text{ gal} = 12 \times 10^4 = \$120,000$$

(c) Find the amount of gasoline it would take to turn the lake to steam (G_s), and its cost C_s .

$$G_s = \frac{Q}{\epsilon} = \frac{3.3 \times 10^{16} \text{ J}}{1.3 \times 10^8 \text{ J/gal}} \approx 2.5 \times 10^8 \text{ gal}$$

$$C_s = \frac{Q}{\epsilon} \cdot P = 2.5 \times 10^8 \text{ gal} \times \frac{\$2}{\text{gal}} = 5 \times 10^8 = \$500 \text{ Million}$$

Lake's worth of water vapor contains much more heat energy than gravitational energy.

Section C: solve these mathematical questions the simplest way possible. Show your work.

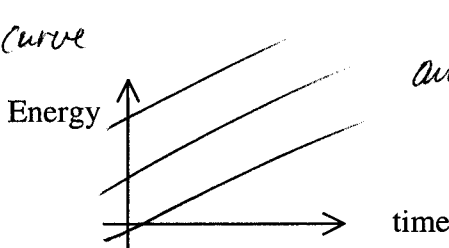
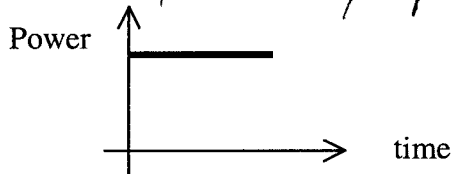
No calculators or computers.

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- 1(a) What is the fundamental relationship between energy and power?
 (b) If the power input to a system is constant and there are no losses, sketch the energy input.
 (c) Explain.

$$\text{Power} = \frac{dE}{dt}$$

Power = Slope of $E(t)$ curve



any line with constant slope - intercept does not affect slope

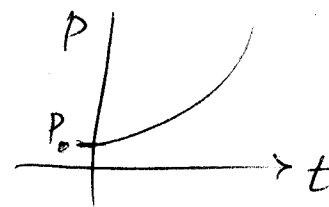
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2. If a population P grows in time t according to $P(t) = P_0 a^t$, explain in words:

- (a) What is P_0 ? (b) What is a ? $P_0 = \text{original population, at } t=0. \quad a = \text{growth}$
 (c) Sketch $P(t)$
 (d) Calculate a if P doubles in 10 years.

$$\frac{P}{P_0} = 2 = a^{10}$$

$$2^{1/10} = a = \sqrt[10]{2}$$



- (e) If instead we write $P(t) = P_0 e^{bt}$, calculate b if P doubles in 10 years.

$$\frac{P}{P_0} = 2 = e^{10b}$$

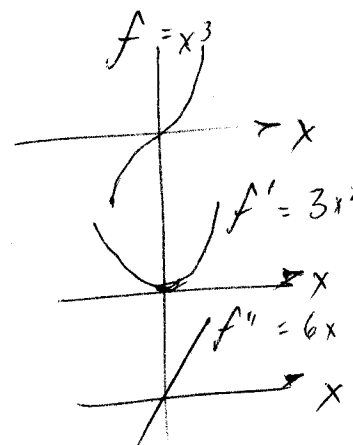
$$\ln 2 = \ln e^{10b} = 10 \cdot b \ln e = 10b \rightarrow b = \frac{\ln 2}{10}$$

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3. Derivatives:

- (a) If $f(x) = x^3$, calculate the derivative: $df/dx = \frac{d}{dx} x^3 = 3x^2$
 (b) Sketch and label f and df/dx
 (c) Calculate and sketch d^2f/dx^2

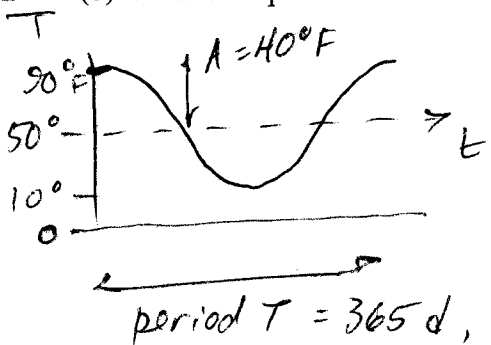
$$\frac{d^2f}{dx^2} = \frac{d}{dx} \frac{df}{dx} = \frac{d}{dx} 3x^2 = 3 \cdot 2x = 6x$$



Rule: $\frac{d}{dx} x^p = px^{p-1}$
 $\frac{d}{dx} x^3 = 3x^{3-1} = 3x^2$

4. If Olympia's weather has a 365-day cycle, with a maximum temperature of 90°F and a minimum of 10°F .

- (a) Sketch the temperature T vs time t , labeling scales on both axes, then
 (b) Write an equation for the temperature vs time, starting at midsummer.

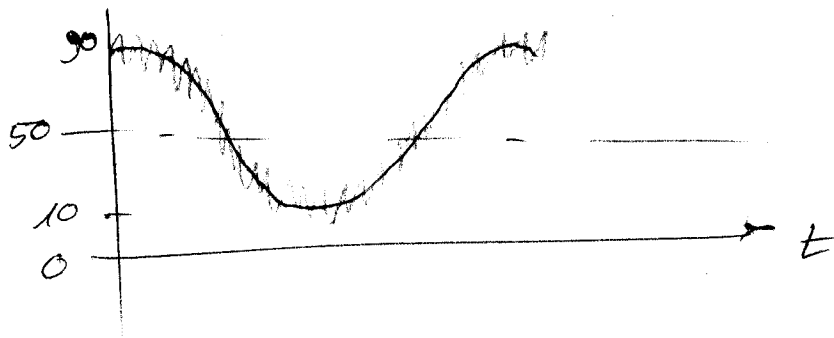


$$T = T_{\text{average}} + A \cos \omega t$$

$$T = 50^\circ\text{F} + 40^\circ\text{F} \cos \frac{2\pi t(\text{d})}{365 \text{ d}}$$

$$\text{period } T = 365 \text{ d}, \quad \omega = \frac{2\pi}{T}$$

Extra:



If I average day by day,

$$T_{(of)} = 50 + 40 \cos \frac{2\pi t_1 (\text{day})}{365 \text{ days}}$$



If I include daily fluctuations of, say 10°F amplitude $= D_0$

~~~~~  $D = D_0 \cos \frac{2\pi t_2 (\text{hr})}{24 \text{ hr}}$

Putting them together:

$$T_{(of)} = 50 + 40 \cos \frac{2\pi t_1 (\text{d})}{365 \text{ days}} + 10 \cos \frac{2\pi t_2 (\text{hr})}{24 \text{ hr}}$$

(In fact, the temperature variation is not this smooth & reproducible.)