# CHARGE TO MASS RATIO EXPERIMENT
(by Dr. Tom Grissom and Dr. E.J. Zita, The Evergreen State College, 14.Feb.2005)

In this experiment, we will measure the charge to mass ratio, e/m, of the electron.
This was first measured by the English physicist J.J. Thomson in 1897.

In the apparatus depicted below, a filament heated by an electric current emits electrons.
These electrons are accelerated by a potential difference V (called the anode voltage) . Some
electrons escape through a slit to form an electron beam.
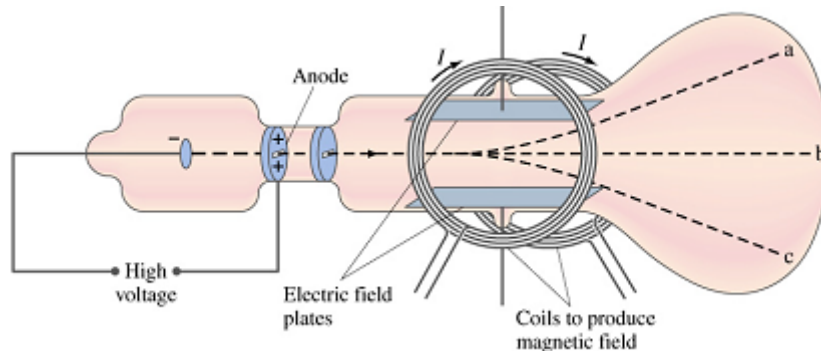
(Figure courtesy of Giancoli, Fig.27-29)

Recall that the work done on a charge q accelerated through a potential difference V is
W=qV. This work increases the charged particle's kinetic energy $K= \frac{1}{2} mv^2$. Equate
$$W=K$$
and solve for the
resultant speed v:

(1) $v =$

Next, the electron beam passes through a gas of mercury (Hg) atoms. When the energy of
the electrons in the beam exceeds the ionization potential of the Hg, colliding electrons can
ionize Hg atoms, causing them to emit light. The path of the electrons through the gas can
then be seen via the emitted light.

If no other forces acted on the beam of electrons, it would continue moving in a straight path
until striking something, in this case the envelope of the glass tube. But the electron tube is
placed between two coils, called Helmholtz coils, which are separated by a distance equal to
the diameter of the coils. Using the Biot Savart law, it can be shown[1] that the magnitude of
the magnetic field in the center of the coils is

$$B = \frac{32\pi \times 10^{-7} N I}{\sqrt{125}\, a}$$

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[1] Giancoli, Physics for Scientists and Engineers 3rd ed, Ch.28 #58,
Halliday, Resnick, and Walker, Fundamentals of Physics 7th edition, Ch.29 #50, 80
Griffiths, Introduction to Electrodynamics 3rd ed., Ch.5 #46, p.239

where I is the current in each coil. In the model of this experiment at TESC, the number of turns in each coil N=72, and the radius of each coil is a=0.33 m. Use these values to find a numerical expression for the magnetic field B in terms of the current I (in SI units):

(2)   B =

Using the right-hand rule for currents, draw the direction of the magnetic field between the two coils. Then use the right-hand rule for the magnetic Lorentz force to sketch the trajectory of the electrons if the magnetic field is turned on.

v
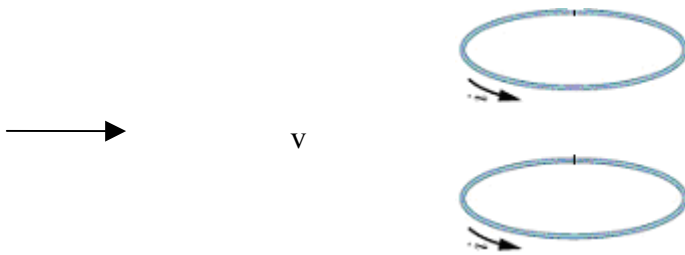
The magnetic field B exerts a force F=qvB on the electrons perpendicular to the direction of their motion, causing them to turn in a circular orbit of radius r (gyroradius) with an unchanged speed. Using the fact that the acceleration is $a=v^2/r$, solve for v:

$$F=ma$$

(3)   v =

Finally, combine equations (1) and (3) to solve for the charge to mass ratio, q/m. (Be careful not to confuse speed v with voltage V.)

(4)   q/m =

Equation 4 gives us a way to measure the charge-to-mass ratio, e/m. By measuring the coil current I (and hence the magnetic field, B, from Eqn.2), the electron's gyroradius r, and the anode voltage V, we can calculate the value of q/m from Eqn.4. The radius of the electron's path is determined by adjusting the magnetic field (the coil current) until the beam hits a series of fluorescing metal bars located known distances away from the beam slit in the electron tube. These measurements provide corresponding values of B and r to use in Eqn.4.

The Earth's magnetic field (B_{Earth}) provides a slight complication. B_{Earth} is nearly, but not quite, vertical at our latitude. By using a compass, we can align the electron tube and the Helmholtz coils until B_{Earth} is along the coils' axis. Then B_{Earth} will be parallel to the applied magnetic field. However, B_{Earth} is down while the coils' field is up. Therefore, with no current in the coils, B_{Earth} deflects the electron beam in the opposite direction.

To correct for this effect, before beginning our measurements, we should turn up the coil current just enough to remove the deflection caused by B_{Earth}. This value of the current (I_1) should then be subtracted from each measured value of the coil current (I_2) to obtain the current ($I=I_2-I_1$) used to find the true value of the deflecting field B.

To operate the apparatus, first set the anode voltage at a convenient value (25 V) before turning the filament current on. Then slowly increase the filament current to about 2.5 A (any higher than this will shorten the filament life) until a beam of electrons can be seen. Next, cancel out the deflection due to B_{Earth} as described above, and note the correction current I_1. Then measure the values of the coil current for which the beam hits each of the fixed posts in the table below. Also note the uncertainty Δ in your measured values.

Cross Bar No.	r (meters)	V (volts)	I_1 (amps)	I_2 (amps)	$I = I=I_2-I_1$ (amps)	B (Tesla)	q/m (coul/kg)
1	0.058						
$\Delta(\pm)$	-						
2	0.052						
$\Delta(\pm)$	-						
3	0.045						
$\Delta(\pm)$	-						
4	0.039						
$\Delta(\pm)$	-						
5	0.033						
$\Delta(\pm)$	-						

Repeat the measurements as you turn the current down, and the beam sweeps across the bars from the opposite direction. How can you fine-tune your measurement technique? Average your measurements for each bar as you turn the current up and down.

Calculate a value of q/m for each bar, and then average all your values.

q/m (measured) =

Finding the uncertainty in q/m:

There is a certain amount of uncertainty in each of your measurements. How far off could the correction current I_1 be off? If takes anywhere between 0.4 and 0.6 amps to eliminate the effects of Earth's field, for example, then $I_1=0.5\pm0.1$ amps, for an uncertainty of $\Delta I_1=0.1$ amps. Similarly, if the current (I_2) it takes to hit the first bar could be anywhere between 0.9 and 1.3 amps, then $I_2=1.1\pm0.2$ amps, for an uncertainty of $\Delta I_2=0.1$ amps. When you calculate $I = I=I_2-I_1$ amps, how far could it be off? The maximum uncertainty in I is $\Delta I_1 + \Delta I_2 = 0.1 + 0.2 = 0.3$ amps. In general, we add uncertainties for added (or subtracted) quantities:
$$\Delta I = \Delta I_1 + \Delta I_2$$

For multiplied (or divided) quantities with different units, such as B=I/k, we can use the fractional uncertainty:
$$\frac{\Delta B}{B} = \frac{\Delta k}{k} + \frac{\Delta I}{I}$$
Note that if k is a constant, then $\Delta k=0$.

Find an algebraic expression for the fractional uncertainty in $q/m=2V/(B^2r^2)$ (Hint: $r^2=r*r$)

(5) $\dfrac{\Delta\left(q/m\right)}{q/m} =$

Then use Eqn.5 to calculate the uncertainty in your value of q/m.

$\Delta\left(q/m\right) =$

Your final answer should be written as q/m ± Δ(q/m) = _____ ± _____ coulombs.

Finding the mass of the electron: Between 1910-1913, American physicist Robert Millikan used his oil drop experiment to measure the charge of the electron. Using the value $q=1.602\times10^{-19}$ coulombs in your expression for q/m, you can now find the mass of the electron. Also calculate the fractional uncertainty Δm.

m_e = _____ ± _____ kg

Finally, compare your values with the accepted values for q/m and the mass of the electron. Does your measurement ± uncertainty span these values?