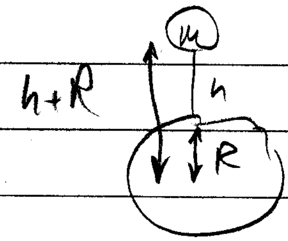
Physics of Astronomy - Phys B - week 3 Zita

Giancoli Ch 6 #6, 11, 13, 27, 36, 41, 42, 62
(you chose these from my candidates)

6. (II) Calculate the effective value of g, the acceleration of gravity, at (*a*) 3200 m, and (*b*) 3200 km, above the Earth's surface.



$F = ma$

$$\frac{GmM}{r^2} = mg' = \frac{GmM}{(h+R)^2}$$

$M = M_{EARTH}$

$M = 6 \times 10^{24}\ kg$

$R = 6.4 \times 10^6\ m$

$$g' = \frac{GM}{(h+R)^2} = \frac{GM}{(h^2 + R^2 + 2hR)}$$

h^2 negligible if $h \ll R$

(a) $h = 3200\ m = 3.2 \times 10^3\ m \ll R = 6.4 \times 10^6\ m$ so I can use

$$g' \approx \frac{GM}{(R^2 + 2hR)}$$

compare to the acceleration g at Earth's surface:

$$g = \frac{GM}{R^2} \longrightarrow \frac{g'}{g} = \frac{R^2}{R^2 + 2hR} = \frac{1}{1 + 2h/R}$$

$$\frac{2h}{R} = \frac{2 \times 3.2 \times 10^3\ m}{6.4 \times 10^6\ m} = \frac{6.4 \times 10^3}{6.4 \times 10^6} = 10^{-3} \rightarrow$$

$$\frac{g'}{g} = \frac{1}{1 + 10^{-3}} \approx 1$$ - very close to $g = 9.8\ m/s^2$

(b) $$\frac{g'}{g} = \frac{GM/(h+R)^2}{GM/R^2} = \left(\frac{R}{h+R}\right)^2 = \left(\frac{1}{h/R + 1}\right)^2 = \left(\frac{1}{\frac{1}{2}+1}\right)^2 = \left(\frac{1}{3/2}\right)^2$$

$$\frac{h}{R} = \frac{3.2 \times 10^6\ m}{6.4 \times 10^6\ m} = \frac{1}{2}$$

$$\frac{g'}{g} = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$ ABOUT HALF

M

11. (II) Suppose the mass of the Earth were doubled, but it kept the same density and spherical shape. How would the weight of objects at the Earth's surface change?

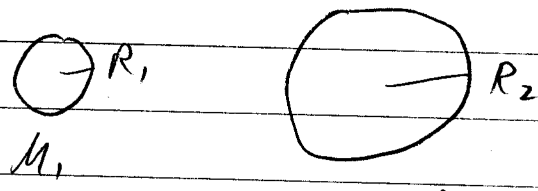
m

$$\text{weight} = mg = \frac{GmM}{R^2}$$

Assume uniform ρ:

$$\rho = m/\text{vol} = \frac{m}{\frac{4}{3}\pi R^3}.$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$



$M_2 = 2M_1$

what is $\frac{R_2}{R_1}$?

Same density: $\rho_1 = \rho_2$

$$\frac{m_1}{\frac{4}{3}\pi R_1^3} = \frac{m_2}{\frac{4}{3}\pi R_2^3} = \frac{2m_1}{\frac{4}{3}\pi R_2^3}$$

$$\left(\frac{R_2}{R_1}\right)^3 = 2$$

$$\text{so } \frac{R_2}{R_1} = 2^{1/3}$$

(The new Earth is a little bigger

$$\frac{\text{new weight}}{\text{old weight}} = \frac{mg_2}{mg_1} = \frac{g_2}{g_1} = \frac{GM_2/R_2^2}{GM_1/R_1^2}$$

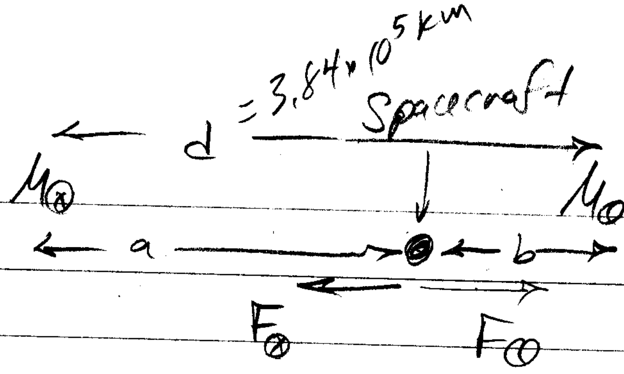
$$= \frac{M_2}{M_1}\left(\frac{R_1}{R_2}\right)^2 = 2\left(\frac{1}{2^{1/3}}\right)^2 = \frac{2^1}{2^{2/3}}$$

$$= 2^{1-2/3} = 2^{1/3}$$

(objects would weigh a little more

a

13. (II) At what distance from the Earth will a spacecraft on the way to the Moon experience zero net force due to these two bodies because the Earth and Moon pull with equal and opposite forces?



$$F_{\otimes} = F_{☾}$$

$$\frac{GmM_{\otimes}}{a^2} = \frac{GmM_{☾}}{b^2}$$

$$a^2 = b^2 \frac{M_{\otimes}}{M_{☾}}$$ and $$d = a + b$$

$$b = d - a$$

$$a^2 = (d-a)^2 \frac{M_{\otimes}}{M_{☾}}$$

$$\frac{M_{☾}}{M_{\otimes}} a^2 = (d^2 - 2ad + a^2) = ka^2$$ where

$$k = \frac{M_{☾}}{M_{\otimes}} = \frac{7.35\times10^{22}\,\text{kg}}{6\times10^{24}\,\text{kg}} = 1.23\times10^{-2} \qquad (1-k) = 0.988$$

$$0 = a^2(1-k) - 2ad + d^2$$

$$a = \frac{+2d \pm \sqrt{(2d)^2 - 4d^2(1-k)}}{2(1-k)} = \frac{2d \pm 2d\sqrt{1-(1-k)}}{2(1-k)}$$

$$\frac{a}{d} = \frac{(1 \pm \sqrt{k})}{1-k} = \frac{(1 \pm \sqrt{1.23\times10^{-2}})}{0.988} = 1.12 \text{ or } 0.90 ✓$$

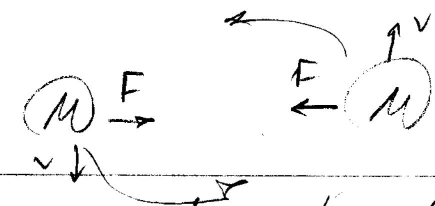
nonsense - past the moon

$$a = 0.90\,(3.84\times10^5\,\text{km}) = 3.46\times10^5\,\text{km}$$

Actually, this is not quite exact due to centripetal forces.

$M = M$ $= T$

$6,$ $2r =$ **27.** (II) Two equal-mass stars maintain a constant distance apart of 8.0×10^{10} m and rotate about a point midway between them at a rate of one revolution every 12.6 yr. (*a*) Why don't the two stars crash into one another due to the gravitational force between them? (*b*) What must be the mass of each star?



(a) Their gravitational attraction keeps them bound in mutual orbit - otherwise they would fly apart, like a spinning ball on a cut string.

(b) $F = \frac{Gmm}{d^2}$ where $d = 2r$ = distance between masses

orbit radius

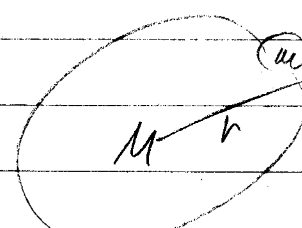
$$F = \frac{Gmm}{(2r)^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2 = \frac{4\pi^2 r^2 m}{r\,T}$$

$$\frac{Gm}{4r^2} = \frac{4\pi^2 r}{T^2} \rightarrow m = \frac{16\pi^2 r^3}{GT^2}$$

$$m = \frac{16\pi^2 (4\times10^{10}\text{ m})^3}{6.67\times10^{-11}\frac{\text{m}^3}{\text{kg s}^2}\left(12.6\text{ yr}\frac{\pi\times10^7\text{ s}}{\text{yr}}\right)^2} = 9.6\times10^{26}\text{ kg}$$

M

36. (I) Determine the mass of the Earth from the known period and distance of the Moon.

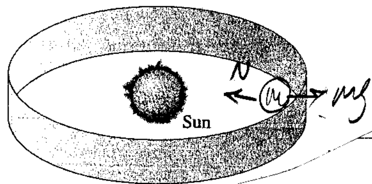


$$\frac{GmM}{r^2} = \frac{mv^2}{r} = \frac{m}{r}\left(\frac{2\pi r}{T}\right)^2$$

$$M = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2(3.84\times10^8\text{ m})^3}{6.67\times10^{-11}\frac{\text{m}^3}{\text{kg s}^2}\left(27\text{ d}\,\Big|\,\frac{3.6\times10^3\text{ s}}{\text{d}}\right)^2}$$

$M_{EARTH} = 6\times10^{24}$ kg

41. (III) A science fiction tale describes an artificial "planet" in the form of a band encircling a sun as shown: Fig. 6–23. The inhabitants live on the inside surface (where it is always noon). Imagine the sun is exactly like our own, that the distance to the band is the same as the Earth–Sun distance (so the climate is temperate), and that the ring rotates quickly enough to produce an apparent gravity of one g as on Earth. What will be the band's period of revolution, this planet's year, in Earth days?



Gravity on Earth: $F = ma$

$$g = \frac{GM_E}{R_E^2} \leftarrow \frac{GmM_E}{R_E^2} = mg$$

A person of mass m feels as if they have a weight mg if the band pushes back on them with a normal force $N = mg$.

$\odot \xleftarrow{r_\odot} \xrightarrow{N}$ (m)

The Sun's gravitational force $F_\odot = \frac{GmM_\odot}{r_\odot^2}$ combines with N to provide the centripetal acceleration.

$$\Sigma F = ma$$

$$F_\odot + N = mv^2/r_\odot$$

$$\frac{GmM_\odot}{r_\odot^2} + mg = \frac{m}{r_\odot}\left(\frac{2\pi r_\odot}{T}\right)^2 = \frac{4\pi^2 m r_\odot}{T^2}$$

$$\frac{GM_\odot}{r_\odot^2} + g = \frac{4\pi^2 r_\odot}{T^2}$$

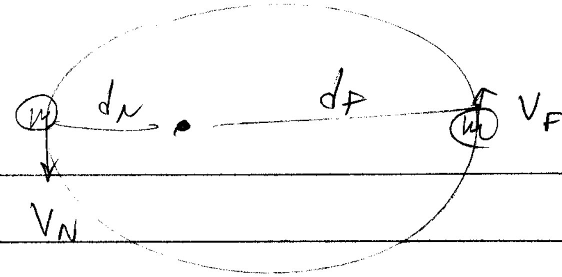
$$\frac{1}{T^2} = \frac{1}{4\pi^2}\left(\frac{GM_\odot}{r_\odot^3} + \frac{g}{r_\odot}\right) \approx \frac{1}{40}\left(\frac{6.67\times10^{-11} \cdot 2\times10^{30}\,kg}{(1.5\times10^{11}\,m)^3} + \frac{9.8\,m/s^2}{1.5\times10^{11}\,m}\right)$$

$$= 1.63\times10^{-12}\ \frac{1}{s^2}$$

$$T = 7.8\times10^5\,s \cdot \frac{hr}{3.6\times10^3\,s} \cdot \frac{d}{24\,hr} = 9\ \text{days}$$

42. (III) (*a*) Use Kepler's second law to show that the ratio of the speeds of a planet at its nearest and farthest points from the Sun is equal to the inverse ratio of the near and far distances: $v_N/v_F = d_F/d_N$. (*b*) Given that the Earth's distance from the Sun varies from 1.47 to 1.52×10^{11} m, determine the minimum and maximum velocities of the Earth in its orbit around the Sun.

d_N d_F



(a) Angular momentum L is conserved, since no external torque operates: $L_N = L_F$ where $L = mVd$

$$mV_N d_N = mV_F d_F$$

$$V_N d_N = V_F d_F \rightarrow \frac{V_N}{V_F} = \frac{d_F}{d_N}$$

(b) This is one equation in two unknowns, so I need one more constraint. $M_\odot = 2 \cdot 10^{30}$ kg

$$F = \frac{GmM}{r^2} = \frac{mv^2}{r} \rightarrow v^2 = \frac{GM}{r}$$

If I solve for V_N, then I can find V_F.

$$V_N = \sqrt{\frac{GM_\odot}{d_N}} = \sqrt{\frac{6.67 \times 10^{-11} \frac{m^3}{kg \cdot s^2} 2 \cdot 10^{30} \text{ kg}}{1.47 \times 10^{11} \text{ m}}} = 3.01 \times 10^4 \frac{m}{s}$$

I could either find $V_F = \sqrt{\frac{GM_\odot}{d_F}}$ or

$$V_F = V_N \frac{d_N}{d_F} = \left(3.01 \frac{m}{s}\right) \frac{1.47}{1.52} = \underline{2.91 \cdot 10^4} \frac{m}{s}$$

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