24. (III) Two equal masses, m_1 and m ...

Physics of Astronomy – week 7 – Giancoli Ch 7 & 8
Phys A
Work & Energy

E/Z

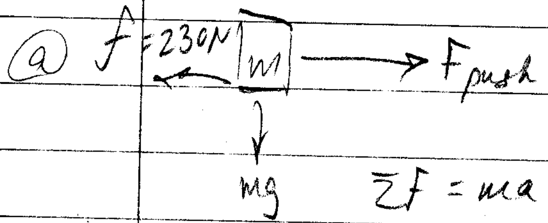
Ch 7 # 4, 6, 21, 34, 39, 41, 52 / Ch 8 # 10, ~~32~~, 45, 47

(b)

4. (I) A 1200-N crate rests on the floor. How much work is required to move it at constant speed (a) 4.0 m along the floor against a friction force of 230 N, and (b) 4.0 m vertically?

$v = const$

$a = 0$

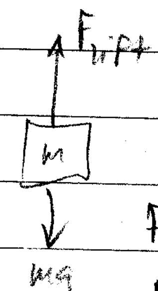


$\Sigma F = ma$

$F - f = 0$

$F = f = 230N$

$W_{(a)} = F \cdot x = 230N \cdot 4m = 920\ N \cdot m$



$\Sigma F = ma$

$F_{lift} - mg = 0$

$F_{lift} = mg$

$W_{(b)} = F_{lift} \cdot x$

$= 1200 N \cdot 4 m$

$= 4800\ Nm$

6. (I) How high will a 1.85-kg rock go if thrown straight up by someone who does 80.0 J of work on it? Neglect air resistance.

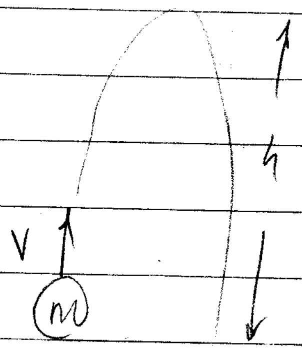
Work done ⇒ initial KE → U at top

$W = \frac{1}{2} m v^2 = mgh$

$W = mgh$

$h = \frac{W}{mg} = \frac{80 J}{1.85 kg \cdot 9.8 m/s^2}$

$h = 4.4\ m$



$A_x \quad A_y \quad B_x \quad B_y \quad B_z$

21. (II) If $\mathbf{A} = 7.0\mathbf{i} - 8.5\mathbf{j}$, $\mathbf{B} = -8.0\mathbf{i} + 8.1\mathbf{j} + 4.2\mathbf{k}$, and $\mathbf{C} = C_x\hat{i} + C_y\hat{j}$ = $6.8\mathbf{i} - 7.2\mathbf{j}$, determine (*a*) $\mathbf{A}\cdot(\mathbf{B}+\mathbf{C})$; (*b*) $(\mathbf{A}+\mathbf{C})\cdot\mathbf{B}$; (*c*) $(\mathbf{B}+\mathbf{A})\cdot\mathbf{C}$.

(a)
$$\vec{B}+\vec{C} = (B_x+C_x)\hat{i} + (B_y+C_y)\hat{j} + (B_z+C_z)\hat{k}$$
$$= (-8+6.8)\hat{i} + (8.1-7.2)\hat{j} + (4.2-0)\hat{k}$$
$$\vec{D} = -1.2\,\hat{i} + 0.9\,\hat{j} + 4.2\,\hat{k}$$

$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{D} = A_xD_x + A_yD_y + A_zD_z$$
$$= 7(-1.2) + (-8.5)0.9 + 0(4.2)$$
$$= -16.1$$

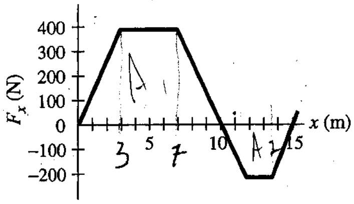
(b)
$$(\vec{A}+\vec{C}) = (A_x+C_x)\hat{i} + (A_y+C_y)\hat{j} + (A_z+C_z)\hat{k}$$
$$= (7+6.8)\hat{i} + (-8.5-7.2)\hat{j} + 0\hat{k}$$
$$\vec{E} = 13.8\,\hat{i} - 15.7\,\hat{j}$$

$$(\vec{A}+\vec{C})\cdot\vec{B} = \vec{E}\cdot\vec{B} = E_xB_x + E_yB_y + E_zB_z$$
$$= 13.8(-8) + (-15.7)8.1 + 0$$
$$= -238$$

(c)
$$(\vec{B}+\vec{A}) = (B_x+A_x)\hat{i} + (B_y+A_y)\hat{j} + (B_z+A_z)\hat{k}$$
$$= (-8+7)\hat{i} + (8.1-8.5)\hat{j} + (4.2+0)\hat{k}$$
$$\vec{F} = -1\,\hat{i} - 0.4\,\hat{j} + 4.2\,\hat{k}$$

$$(\vec{B}+\vec{A})\cdot\vec{C} = \vec{F}\cdot\vec{C} = F_xC_x + F_yC_y + F_zC_z$$
$$= (-1)6.8 + (-0.4)(-7.2) + 0$$
$$= -3.9$$

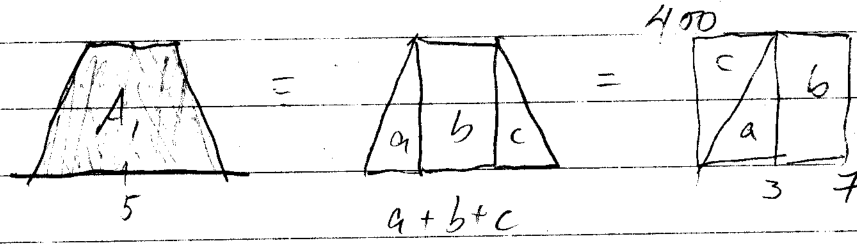
34. (II) The force on a particle, acting along the x axis, varies as shown in Fig. 7–26. Determine the work done by this force to move the particle along the x axis: (a) from $x = 0.0$ to $x = 10.0$ m; (b) from $x = 0.0$ to $x = 15.0$ m.

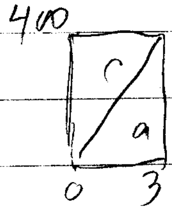


Work $= \int F \cdot dx$

$=$ area under curve

$= A_1 - A_2$

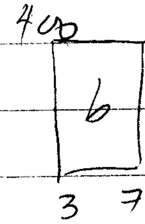




$a + c = 3 \times 400 = 1200$ N·m

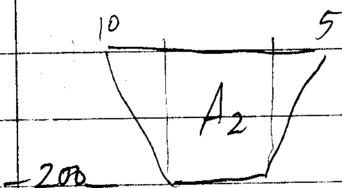
$b = 400(7-3) = 1600$ Nm

$a + b + c = A_1 = \underline{2800 \text{ Nm}}$



(a) This is the work done to $x = 10$ m

(b) Between $x = 10 - 15$ m, the force is AGAINST the motion. So NEGATIVE work A_2 is done.



Note that this is the same shape as A_1 except base and height are half so

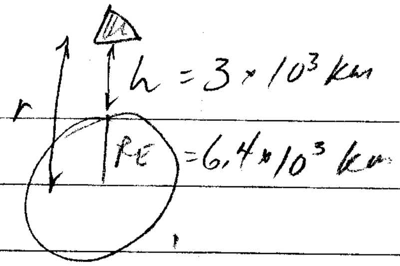
$A_2 = \frac{1}{2} \cdot \frac{1}{2} A_1 = \frac{1}{4} A_1$

So up to $x = 15$ m, Work $= A_1 - A_2 = A_1 - \frac{1}{4} A_2$

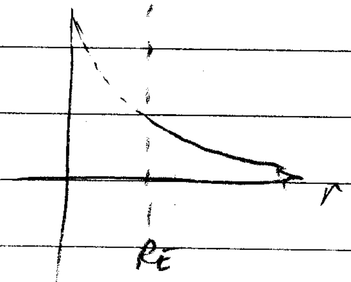
$= \frac{3}{4} A_2 = \frac{2800}{4} \cdot 3$

$W = 700 \cdot 3 = 2100$ N·m

39. (III) A 2500-kg space vehicle, initially at rest, falls vertically from a height of 3000 km above the Earth's surface. (*a*) Determine how much work is done by the force of gravity in bringing the vehicle to the Earth's surface by first constructing an F versus r graph (using Eq. 7–1), where r is the distance from the Earth's center. Then determine the work graphically to an accuracy of 3 percent. (*b*) Repeat using integration.



$$r = h + R_E \approx \tfrac{1}{2}R_E + R_E \approx \tfrac{3}{2}R_E$$



$$F(r) = \frac{GmM}{r^2}$$

Work done = Decrease in U = Increase in K

$$U = -\int_{R_E}^{3R_E/2} F\,dr = -\int \frac{GmM}{r^2}\,dr = GmM\left(-\frac{1}{r}\right)\Big|^{3R_E/2}$$

$$= -GmM\left(\frac{1}{3R_E/2} - \frac{1}{R_E}\right) = \frac{-GmM}{R_E}\left(\frac{2}{3} - \frac{3}{3} = -\frac{1}{3}\right)$$

$$U = \frac{1}{3}\frac{GmM}{R_E} \qquad \text{Note that } F(R_E) = mg = \frac{GmM}{R_E^2}$$

$$g = \frac{GM}{R_E^2}$$

$$U = \frac{1}{3}mgR_E$$

$$= \frac{1}{3}(2500\text{ kg})(10\tfrac{m}{s^2})(6.4\times10^6\text{ m}) = \underline{5.3\times10^{10}}\text{ Joules}$$

41. (I) (*a*) If the kinetic energy of a particle is tripled, by what factor has its speed increased? (*b*) If the speed of a particle is halved, by what factor does its kinetic energy change?

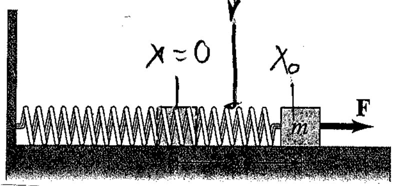
$$K = \tfrac{1}{2}mv^2 \rightarrow v^2 = \frac{2K}{m}$$

If $K_2 = 3K_1$, find $\left(\frac{v_2}{v_1}\right)^2 = \frac{2K_2/m}{2K_1/m} = \frac{K_2}{K_1} = 3$

$\underline{v_2 = \sqrt{3}\, v_1}$

(b) $\frac{K_2}{K_1} = \left(\frac{v_2}{v_1}\right)^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

52. (II) A mass m is attached to a spring which is held stretched a distance x_0 by a force F (Fig. 7–28), and then released. The spring compresses, pulling the mass. Assuming there is no friction, determine the speed of the mass m when the spring returns: (*a*) to its normal length ($x = 0$); (*b*) to half its original extension ($x_0/2$).



$E_{tot} = U_{stretched} = \frac{1}{2}kx_0^2 = U_0$

At any point x,

$E_{tot} = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$F = kx \rightarrow k = F/x$

(a) When spring returns to $x=0$, all the potential energy U_0 has converted to kinetic energy: $U(x=0) = \frac{1}{2}k0^2 = 0$

$\frac{1}{2}mv^2 = \frac{1}{2}kx_0^2$

$v^2 = \frac{kx_0^2}{m} \rightarrow v = x_0\sqrt{\frac{k}{m}} = x_0\sqrt{\frac{F/x_0}{m}} = \sqrt{\frac{Fx_0}{m}}$

(b) When spring returns to $x = \frac{x_0}{2}$,

$E_{tot} = K + U$

$\frac{1}{2}kx_0^2 = \frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{x_0}{2}\right)^2$

$kx_0^2 = mv^2 + \frac{1}{4}kx_0^2$

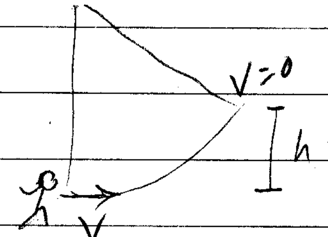
$kx_0^2(1-\frac{1}{4}) = \frac{3}{4}kx_0^2 = mv^2$

$v = x_0\sqrt{\frac{3k}{4m}} = \sqrt{\frac{3}{4}\frac{Fx_0}{m}}$

Giancoli

Ch 8 # 10, 32, 45, 47

10. (I) Jane, looking for Tarzan, is running at top speed (5.0 m/s) and grabs a vine hanging 4.0 m vertically from a tall tree in the jungle. How high can she swing upward? Does the length of the vine (or rope) affect your answer?



$K_{top} = 0$

$U_{top} = mgh$

$K_i = \frac{1}{2}mv^2$

$U_i = 0$

$E_{initial} = E_{top}$

$K_i + U_i = K_{top} + U_{top}$

$\frac{1}{2}mv^2 = mgh$

$h = \frac{v^2}{2g} = \frac{(5\,m/s)^2}{2 \cdot 10\,\frac{m}{s^2}} = \frac{25}{20} = \frac{5}{4}\,m$

(Length of vine is irrelevant)

32. (II) Consider the track shown in Figure 8–32. The section AB is one quadrant of a circle of radius 2.0 m and is frictionless. B to C is a horizontal span 3.0 m long with a coefficient of kinetic friction $\mu_k = 0.25$. The section CD under the spring is frictionless. A block of mass 1.0 kg is released from rest at A. After sliding on the track, it is observed to compress the spring by 0.20 m. Determine: (*a*) the velocity of the block at point B; (*b*) the work done by friction as the block slides from B to C; (*c*) the velocity of the block at point C; (*d*) the stiffness constant *k* for the spring.

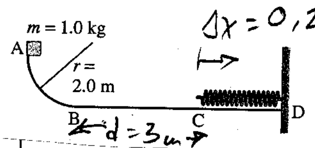


FIGURE 8–32 Problem 32.

– NOT REQUIRED –

$U_A = K_B = \frac{1}{2}k\Delta x^2 + W_F$

energy lost to friction

$W_F = \mu mg \cdot d$

$U_A = K_B$

(a) $mgh = \frac{1}{2}mV_B^2 \longrightarrow V_B^2 = 2gh \simeq 2(10\,\frac{m}{s^2})\,2m = 2^2 \times 10\,\frac{m^2}{s^2}$

$V_B = 2\sqrt{10} \gtrsim 6\,m/s$

(b) $W_F = F_{friction} \cdot distance = \mu mg \cdot d = \frac{1}{4} \cdot 1kg \cdot 10\,\frac{m}{s^2} \cdot 3m = \frac{3 \cdot 5}{2}\,J$

$W = 7.5\,J$

(c) $U_A - W_F = K_c$

$mgh - \mu mg d = \frac{1}{2}mV_c^2 \rightarrow V_c^2 = 2g(h - \mu d)$ $\quad V_c = 5\,\frac{m}{s}$

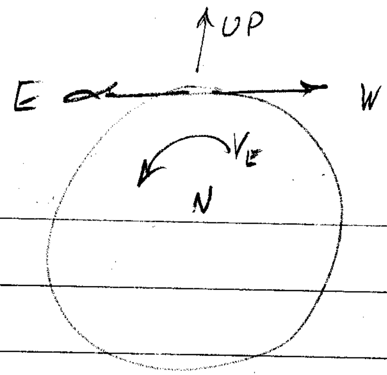
$V_c^2 \simeq 2 \cdot 10\,\frac{m}{s^2}(2m - \frac{1}{4} \cdot 3m = 1\frac{1}{4} = \frac{5}{4}\,m) = \frac{20}{4} \cdot 5 = 25\,\frac{m^2}{s^2}$

(d) If $\boxed{m}$ stops after compressing spring by Δx, then

$K_c = \frac{1}{2}k\Delta x^2 = \frac{1}{2}mV_c^2 \rightarrow k = m\left(\frac{V_c}{\Delta x}\right)^2 = 1kg\left(\frac{5\,m/s}{1/5\,m}\right)^2 = 25^2\,N/m$ $\quad 625$

G8*

45. (II) Take into account the Earth's rotational sp (1 rev/day) and determine the necessary speed, with res to Earth, for a rocket to escape if fired from the Earth at equator in a direction (*a*) eastward; (*b*) westward; (*c*) v cally upward.



Sun rises East → west

So Earth turns west → east

What is the speed of the earth at the equator?

$$V_{EARTH} = V_E = \frac{2\pi R}{T} = \frac{2\pi \times 6.4\times10^6\,m}{day}\left|\frac{24\,hr}{d}\right|\frac{3.6\times10^3\,s}{hr}\bigg| \approx \frac{7\times6}{6\times4\times3.6}\times10^{6-3}\,\frac{m}{s} \approx \frac{2}{4}\times10^3$$

$$V_E = 465\,\frac{m}{s}$$

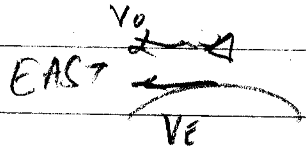
What is the escape speed, ignoring Earth's motion?
Last week we derived this: $K = U$

$$\frac{1}{2} m V_e^2 = \frac{GmM_E}{R_E}$$

$$V_{escape} = V_e = \left(\frac{2GM}{R_E}\right)^{\frac{1}{2}} = \left(\frac{2\cdot 6.67\times10^{-11}\,\frac{m^3}{kg\,s^2}\,6\times10^{24}\,kg}{6.4\times10^6\,m}\right)^{\frac{1}{2}} \gtrsim \left(1.2\times10^{24-11-6}\,\frac{m^2}{s^2}\right)^{\frac{1}{2}}$$

$$= \sqrt{120\times10^{7-1}}\,\frac{m}{s} \simeq 11\times10^{6/2=3}\,\frac{m}{s} = 11183\,\frac{m}{s} = 1.12\times10^4\,\frac{m}{s}$$

(a) Consider a rocket fired EASTward with a speed V_0 RELATIVE to Earth. It's like throwing a ball out a car window; the ball goes faster if the car is moving.

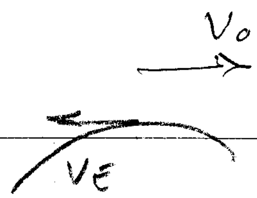


$V_0 + V_E$ effective speed

$V_{escape} = V_e = V_0 + V_E$ → Solve for V_0 = rocket speed.

$$V_0 = V_e - V_E = 1.12\times10^4 - 465 = 1.07\times10^4\,m/s$$

45 (b) Consider a rocket launched WESTWARD

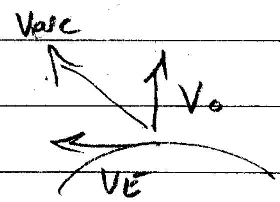


The rocket must overcome Earth's motion

$V_{escape} = V_0 - V_E$ → Solve for V_0

$V_0 = V_e + V_E \approx 1.12\times10^4 + 465 = 1.16\cdot10^4\ m/s$

(c) Consider a rocket launched UP



$V_{escape}^2 = V_E^2 + V_0^2$. Solve for V_0.

$V_0^2 = V_e^2 - V_E^2 = (1.12\cdot10^4\ \frac{m}{s})^2 - (465\ \frac{m}{s})^2 = 1.25\times10^8$

$V_0 = 1.12\cdot10^4\ m/s$ – changed, but not within our precision

47. (II) (*a*) Determine the rate at which the escape velo[city] from the Earth changes with height above the Earth's s[ur]face, dv_{esc}/dr. (*b*) Use the approximation $\Delta v \approx (dv/dr)\,\Delta r$ to determine the escape velocity for a spacecraft orbit[ing] the Earth at a height of 300 km. $= \Delta r$

Re-derive escape speed:

$$K = U$$

$$\frac{1}{2} m v_e^2 = \frac{G m M}{R_E}$$

$$V_e^2 = \frac{2GM}{R_E}$$

In general, at some distance r from Earth's center

$$V_e(r) = \left(\frac{2GM}{r}\right)^{1/2} = \sqrt{2GM}\; r^{-1/2}$$

(a) $$\frac{\partial V_e}{\partial r} = \sqrt{2GM}\;\frac{1}{2}\, r^{-3/2} = \sqrt{\frac{GM}{2r^3}}$$

(b) We know from #85 that $V_e = 1.12 \times 10^4$ m/s at Earth's surface ($r = R_E$). Above the surface

$V(r) \approx V_e - \Delta V$ where $\Delta V \approx \Delta r \dfrac{\partial V_e}{\partial r}$

$$r = R_E + \Delta r = 6.4 \times 10^6\,\text{m} + 3 \times 10^5\,\text{m} = 6.7 \times 10^6\,\text{m}$$

$$\frac{\partial V_e}{\partial r} = \sqrt{\frac{GM}{2r^3}} = \sqrt{\frac{6.67 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}\; 6 \times 10^{24}\,\text{kg}}{2 \times (6.7 \times 10^6\,\text{m})^3}} = 8.2 \times 10^{-4}\,\frac{\text{m/s}}{\text{m}}$$

$$\Delta V = \Delta r \frac{\partial V_e}{\partial r} = 300 \times 10^3\,\text{m} \times 8.2 \times 10^{-4}\,\frac{1}{\text{s}} = 245\,\text{m/s}$$

$$V \approx V_e - \Delta V = 1.12 \times 10^4 - 245 = 1.09 \times 10^4\,\text{m/s}$$