

Part Ch 11.1 # 2, 4, 5, 6

11.2 In three-dimensional space, a classical particle moves in a potential field given by

$$U = V(x, y, z) = ax^3 + by^3 + cz^3 + dxy + exz + fyz,$$

where  $a, b, c, d, e,$  and  $f$  are constants.

(A) When the particle is at the point  $x = 1$  cm,  $y = 2$  cm, and  $z = 3$  cm, what is the  $x$  component of force acting on it in terms of the constants of the problem?

$$\vec{F} = -\vec{\nabla}U = -\frac{\partial U}{\partial x} \hat{i} - \frac{\partial U}{\partial y} \hat{j} - \frac{\partial U}{\partial z} \hat{k}$$

$$F_x =$$

$$F_y =$$

$$F_z =$$

$$F_x(x=1, y=2, z=3) =$$

$$F_y(1, 2, 3) =$$

$$F_z(1, 2, 3) =$$

Let  $a = b = c = 1 \text{ dyne cm}^{-2}$   
and  $d = e = f = 1 \text{ dyne cm}^{-1}$ .

They find, at  $P = (1, 2, 3)$

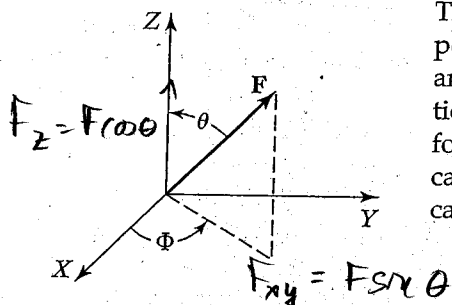
$$F_x(P) =$$

$$F_y(P) =$$

$$F_z(P) =$$

11.2

(B) Consider a vector  $F$  as shown in the following diagram:



The direction of  $F$  may be specified by its spherical polar angles  $\theta$  and  $\phi$ , where  $\theta$  is the angle between  $F$  and the  $z$ -axis and  $\phi$  is the angle between the projection of  $F$  into the  $(x-y)$  plane and the  $x$ -axis. If  $F$  is the force vector on a particle at the point given in (A), calculate the direction of  $F$  by computing the spherical polar angles  $\theta$  and  $\phi$ .

Use  $F_x(P)$ ,  $F_y(P)$ ,  $F_z(P)$   
from Part (A)

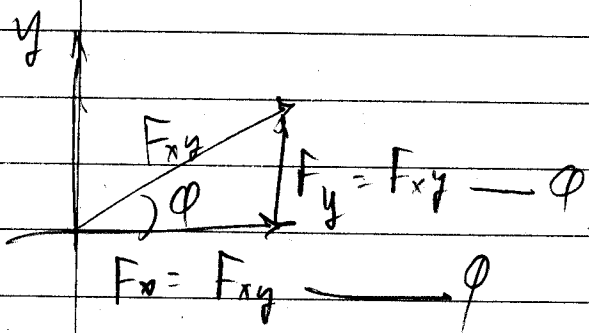
$$F = F_x^2 + F_y^2 + F_z^2$$

$$=$$

$$F_z = F \cos \theta \rightarrow \cos \theta =$$

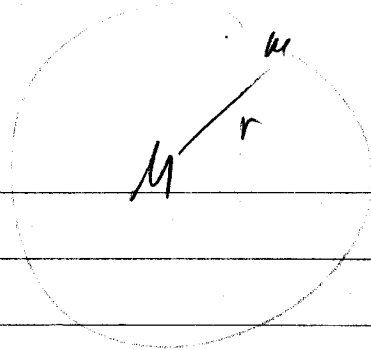
$$\theta =$$

In the  $(x-y)$  plane,  $F_{xy} = F \sin \theta =$



Find  $\phi$ :

(11.4) Assume that the hydrogen atom behaves like a planetary system, with the electron rotating about an extremely heavy nucleus. The mass of the electron is  $m = 9.1094 \times 10^{-31}$  kg. The kinetic energy of the electron  $K = 2.1792 \times 10^{-18}$  J. If the nuclear mass is assumed to be infinite and the orbit circular with a radius of  $r = 0.519 \times 10^{-10}$  m, compute the angular momentum of the electron.



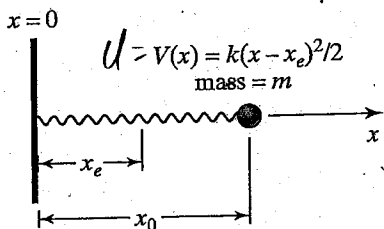
$$(11.22) \quad T = \frac{p_R^2}{2m} + \frac{p_\theta^2}{2mR^2}$$

Why is  $p_R = 0$ ?

$$p_\theta =$$

11.6

Consider a single particle of mass  $m$  attached to a wall of infinite mass by a spring as shown in the following diagram:



The particle moves in one dimension ( $x$ ). The potential for the system is  $V(x) = k(x - x_e)^2/2$ , where  $x_e$  is a constant. At time  $t = 0$ , the particle is at position  $x_0$  and its velocity is zero.

(A) Set up and solve the Newtonian equations of motion for this system. Plot the position of the particle as a function of time from  $t = 0$  to  $t = 4\pi(m/k)^{1/2}$ . Make the abscissa of the plot be in units of  $4\pi(m/k)^{1/2}$  so that the abscissa will go from 0 to 1. (Hint: Refer to Problem 9.5 for assistance in solving the differential equation.)

(B) As can be seen from the plot made in (A), the classical motion of the spring is oscillatory. In fact, this system is called a harmonic oscillator. The period  $\tau$  of the oscillator is the time required to execute one complete vibrational cycle. The vibrational frequency  $\nu$  is the reciprocal of the period. Determine the vibrational period and frequency of this harmonic oscillator in terms of  $k$  and  $m$ .

$$H = T + U = \frac{p_x^2}{2m} + \frac{1}{2}k(x - x_e)^2$$

$$\frac{\partial H}{\partial p} = \frac{\partial x}{\partial t} = \frac{\partial}{\partial p_x} \left( \frac{p_x^2}{2m} \right) = \frac{p_x}{m} = v_x$$

$$\frac{\partial H}{\partial x} = -\frac{\partial p}{\partial t} = \frac{\partial}{\partial x} \left( \frac{1}{2}k(x - x_e)^2 \right) = kx$$

$$F = ma$$

$$-\frac{\partial U}{\partial x} = m \frac{d^2 x}{dt^2} =$$

$$\text{let } z = (x - x_e)$$

$$\frac{dz}{dt} =$$

(Solved in class)

11.5

Assume the particle illustrated in Figure 11.5 is moving in three-dimensional space. Then, in rectangular Cartesian coordinates, the kinetic energy of the particle is

$$T = \frac{m}{2} [v_x^2 + v_y^2 + v_z^2].$$

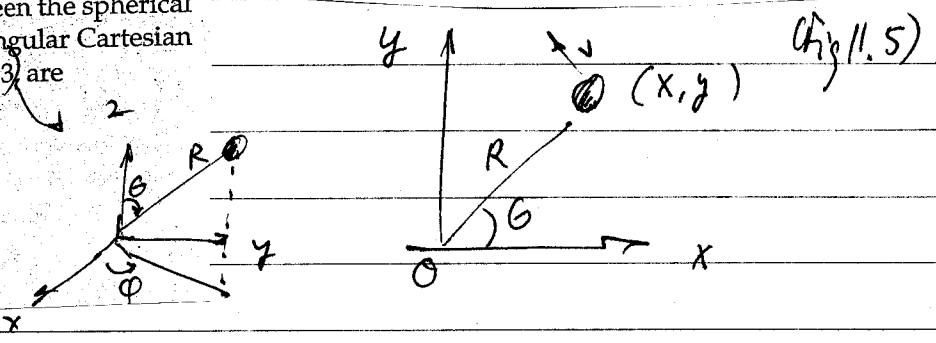
Suppose we wish to transform to a spherical polar coordinate system with coordinates  $(R, \theta, \phi)$  instead of  $(x, y, z)$ . This coordinate system is illustrated in the figure given in Problem 11.2 if we replace  $F$  with  $R$ . The transformation equations between the spherical polar coordinate system and a rectangular Cartesian system, given in Chapter 9 (Figure 9.3) are

$$x = R \sin \theta \cos \phi,$$

$$y = R \sin \theta \sin \phi,$$

$$z = R \cos \theta.$$

and



(A) Obtain the kinetic energy of the particle in terms of the spherical polar coordinates and their rates of change with time.

(B) Obtain expressions for the momenta conjugate to the spherical polar coordinates.

(C) Show that the momentum conjugate to  $\phi$ ,  $P_\phi$ , is the same as the Cartesian z-component of angular momentum.

(D) Express the kinetic energy in terms of the spherical polar conjugate momenta and coordinates.

$$\textcircled{A} \quad V_x = \frac{dx}{dt} = \frac{dR}{dt} \sin \theta \cos \phi + R \cos \phi \frac{d\theta}{dt} \sin \theta + R \sin \theta \frac{d\phi}{dt} \cos \theta$$

$$V_y =$$

$$V_z =$$

Simplify!  $V_x^2 + V_y^2 + V_z^2 =$

$$T =$$

(B) Momenta conjugate to  $(r, \theta, \phi)$  are  $(p_r, p_\theta, p_\phi)$ :

$$p_r = \frac{\partial T}{\partial v_r} =$$

$$p_\theta = \frac{\partial T}{\partial v_\theta} =$$

$$p_\phi = \frac{\partial T}{\partial v_\phi} =$$

(C) Angular momentum about z axis is  $L_z = M_z = p_\phi$

$$\vec{L} = \vec{r} \times \vec{p} = \vec{M} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ v_x & v_y & v_z \end{vmatrix} = \hat{i}(y v_z - z v_y) - \hat{j}(x v_z - z v_x) + \hat{k}(x v_y - y v_x)$$

$$L_z = M_z = (x v_y - y v_x) \\ = R \sin \theta \cos \phi (v_y) - R \sin \theta \sin \phi (v_x)$$

Insert  $v_y$  &  $v_x$ :

Simplify:

Compare to  $p_\phi$ :

① Express  $T$  in terms of  $(r, \theta, \phi)$  and  $(p_r, p_\theta, p_\phi)$

$$\frac{1}{2} m v_r^2 = \frac{p_r^2}{2m}, \quad \frac{1}{2} m (R v_\theta)^2 = \frac{p_\theta^2}{?}$$

$$\frac{1}{2} m (R v_\phi \sin \theta)^2 = \frac{p_\phi^2}{?}$$

$$T = \frac{1}{2} m [v_r^2 + (R v_\theta)^2 + (R v_\phi \sin \theta)^2]$$

$$T = \frac{1}{2} m$$