Physics of Astronomy - week 7 - Ralf Gr 1 E/Z
Phys B #7, 8, 9

11.7 Show that Planck's equation for the energy density of blackbody radiation reduces to the equation suggested by Wien for small values of λT.

Let $a = 8\pi hc$
$b = \frac{hc}{kT}$

(11.29) 556 $u_{planck} = \frac{8\pi hc}{\lambda^5} \frac{1}{\left(e^{hc/\lambda kT} - 1\right)} = \frac{a}{\lambda^5} \frac{1}{e^{b/\lambda} - 1}$

(11.27) Show that $\lim_{\lambda \to 0} u_p = u_{WIEN} = \frac{c_1}{\lambda^5} e^{-c_2/\lambda T}$

$\lim_{\lambda \to 0} \left(e^{hc/\lambda kT} - 1\right) = e^{hc/\lambda kT}$ which becomes much larger than 1.

So $\lim_{\lambda \to 0} u_p =$

$a = 8\pi hc$

$b = \frac{hc}{KT}$

(11.8) Show that Planck's equation for the energy density of blackbody radiation reduces to the equation suggested by Rayleigh and Jeans for large values of λT. (*Hint*: Expand the exponential in a power series.)

(11.29) $$u_{planck} = u_p = \frac{8\pi hc}{\lambda^5} \frac{1}{(e^{hc/k T\lambda} - 1)} = \frac{a}{\lambda^5} \frac{1}{(e^{b/\lambda} - 1)}$$

(11.28) Show that $\lim_{\lambda \to \infty} u_p = u_{RJ} = \frac{8\pi k_b T}{\lambda^4}$ (Rayleigh - Jeans)

We derived the power series expansion for e^x in class:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!}$$

$$(e^{b/\lambda} - 1) = e^x - 1 = (1 + x + \frac{x^2}{2!} + \dots) - 1 = x + \frac{x^2}{2!} + \dots$$

where $x = \frac{hc}{\lambda k T}$. $\lim_{\lambda \to \infty} e^{b/\lambda} =$

Then $\lim_{\lambda \to \infty} u_p =$

(11.9) Show that Planck's equation predicts a maximum in the energy density of blackbody radiation at the point $\lambda \approx 0.290$ cm K/T when $u(T, \lambda)$ is plotted against λ. [*Hint*: You may solve the problem either by a one-dimensional grid search or by iterative methods.]

$$u = \frac{a}{\lambda^5} \frac{1}{(e^{b/\lambda} - 1)}$$

$$a = 8\pi hc, \quad b = hc/kT$$

Extrema in u when $\frac{\partial u}{\partial \lambda} = 0$

Differentiate, set to 0, solve for λ:

$$\frac{\partial u}{\partial \lambda} = \frac{a}{\lambda^5} \frac{\partial}{\partial \lambda}\left(\frac{1}{e^{b/\lambda} - 1}\right) + \frac{1}{(e^{b/\lambda} - 1)} \frac{\partial}{\partial \lambda} \frac{a}{\lambda^5}$$

$$\frac{d}{d\lambda} f^p(\lambda) = p f^{p-1}(\lambda) \frac{\partial}{\partial \lambda} f(\lambda), \quad \text{Let } f(\lambda) = (e^{b/\lambda} - 1)^{-1}$$

$$\frac{\partial}{\partial \lambda}(e^{b/\lambda} - 1)^{-1} =$$