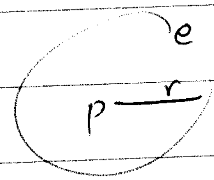
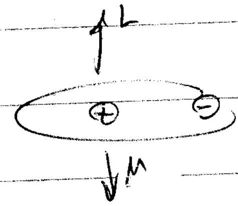
Finishing modern physics - week 8

Angular momentum, magnetic moment, Zeeman effect

BOHR ATOM





$F = ma$

27 Ex 9 / 697 $\quad \frac{kqQ}{r^2} = \frac{e^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \longrightarrow$ Solve for v

Current $I = \frac{dq}{dt} = \frac{e}{\text{period}}$ $\quad$ period $= T = \frac{\text{circumference}}{v} = \frac{2\pi r}{v}$

$I = \frac{ev}{2\pi r}$

Area of e^- orbit $A = \pi r^2$

MAGNETIC MOMENT $\mu \equiv IA = \frac{ev}{2\pi r} \pi r^2 = \frac{evr}{2}$

40.7 / 1015 Write this using ANGULAR MOMENTUM $L = m_e v r \rightarrow vr = \frac{L}{m_e}$

Solve for vr, plug into $\mu = \frac{e}{2}\frac{L}{m_e}$

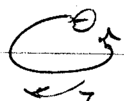
QUANTIZATION of orbital angular momentum : $L = m_\ell \hbar$

new quantum #

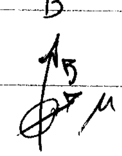
Write μ in terms of m_ℓ : $\mu = \frac{e}{2}\frac{m_\ell \hbar}{m_e} = -m_\ell \mu_B$

BOHR MAGNETON $\mu_B = \frac{e\hbar}{2m_e}$

Bohr atom: electron's orbit → magnetic dipole
(DRAW B FIELD)



Magnetic dipole experiences TORQUE in a magnetic field



Potential energy $U = -\bar{\mu}\cdot\bar{B}$

Zeeman effect: energy levels SPLIT by $\Delta U = \mu_B B$

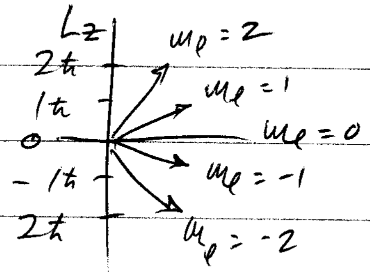
40-2 / 1005 Orbital angular momentum QUANTUM NUMBER $l = 0, 1, \ldots, (n-1)$

magnetic quantum number m_l indicates DIRECTION of l

Orbital angular momentum $L = \sqrt{l(l+1)}\,\hbar$

For B_z, $L_z = m_l \hbar$

If $l = 2$, then m_l can be $0, \pm1$, or ±2.



Do 40#46 / 1027

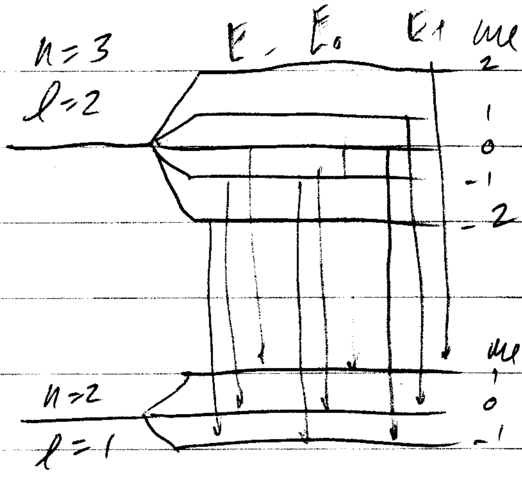
$l=0$	$l=1$	$l=2$	$l=3$
s	p	d	f
0	1	2	3
0	1	2	3
0	1	2	3

MNEMONIC for Filling order:
s0, p1, p0, d2, p1, s0

$$\mu_B = \frac{eh}{2m_e} = \frac{1.6\times10^{\ldots}\ \ldots\times 1.0\ldots}{2\cdot 9.11\times10^{-31}\ \text{kg}} = 9.266\times10^{-24}\ \frac{J}{T}$$

45. (I) Verify that the Bohr magneton has the value $\mu_B = 9.27\times10^{-24}$ J/T (see Eq. 40–12).

46. (II) Suppose that the splitting of energy levels shown in Fig. 40–4 was produced by a 2.0-T magnetic field. (*a*) What is the separation in energy between adjacent m_l levels for the same l? (*b*) How many different wavelengths will there be for 3d to 2p transitions, if m_l can change only by ± 1 or 0? (*c*) What is the wavelength for each of these transitions?



ⓐ $U = -\mu_z B = -\mu_B m_l B$

$\Delta U = \Delta m_l\ \mu_B B$

$= 1.2\times10^{-4}$ eV

ⓑ Transitions are allowed from $\Delta m_l = 0, \pm 1$

40-2
1005

ⓒ $\frac{1}{\lambda_0} = R\left[\frac{1}{n^2} - \frac{1}{n'^2}\right] = 1.0974\times10^{7}\frac{1}{m}\left(\frac{1}{4}-\frac{1}{9}\right)$ $n=2,\ n'=3$

$\lambda_0 = \underline{656.10\ \text{nm}}$

$E = \frac{hc}{\lambda}$, $\frac{dE}{d\lambda} = hc\frac{d}{d\lambda}(\lambda^{-1}) = hc\left(\frac{-1}{\lambda^2}\right)$

$\frac{\Delta E}{E} = -\frac{\Delta\lambda}{\lambda}$ ⟷ $\Delta E = -\frac{hc}{\lambda}\frac{\Delta\lambda}{\lambda} = \mu_B B = 9.27\times10^{-24}\frac{J}{T}(2T)$

$\Delta E = 1.85\times10^{-23}$ J $= 1.16\times10^{-4}$ eV

$E_0 = \frac{hc}{\lambda_0} = \frac{6.63\times10^{-34}\ \text{J·s}\times3\times10^{8}\ \frac{m}{s}}{\lambda_0} = \frac{3.03\times10^{-19}\ \text{J}\cdot\text{m}}{\text{m}} = \underline{1.895\ \text{eV}}$

$E_0+\Delta E = E_+ = \frac{hc}{\lambda_0-\Delta\lambda} = \frac{hc}{\lambda_-}$ or $\frac{\Delta\lambda}{\lambda_0} = \frac{\Delta E}{E} = 6.12\times10^{-5} \rightarrow \lambda_+ = \left(1+\frac{\Delta\lambda}{\lambda}\right)\lambda_0$

$= 656.14$ nm

$E_0-\Delta E = E_- = \frac{hc}{\lambda+\Delta\lambda} = \frac{hc}{\lambda_+}$ $\lambda_- = \left(1-\frac{\Delta\lambda}{\lambda}\right)\lambda_0 = 656.06$ nm

39-3
981

Heisenberg Uncertainity principle

Uncertainity in position $\Delta x \approx \lambda$ of light used to look at particle.

Photon can change particle's momentum by $\Delta p \approx \frac{h}{\lambda}$

$$\Delta x \Delta p \approx \_\_\_\_\_\_$$

$$\Delta x \Delta p \gtrsim \hbar = \frac{h}{2\pi}$$ Do $\frac{\#7}{1000}$

The more precisely we measure position (with light of small λ) the less precisely we can know momentum.

Time uncertainty $\Delta t \approx \frac{\Delta x}{c} \approx \frac{\lambda}{c}$ (travel time of photon across object)

Photon can change particle's energy by $\Delta E \approx \frac{hc}{\lambda}$

$$\Delta E \, \Delta t \approx \_\_\_\_\_\_$$

$$\Delta E \, \Delta t \gtrsim \hbar$$ Do $\frac{\#9}{1000}$