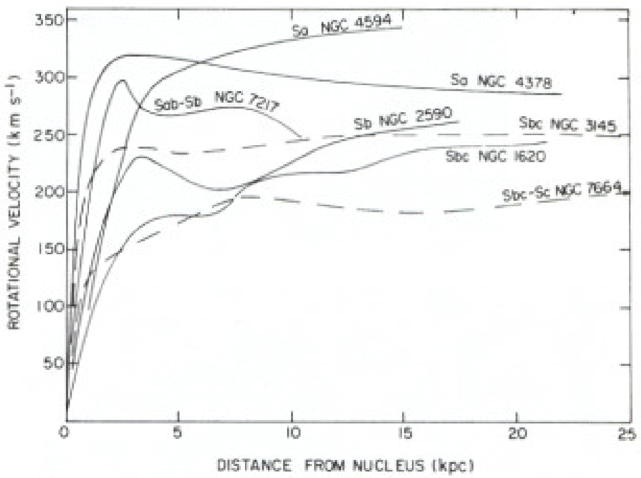


**Figure 22.29** A series of rotation curves for spiral galaxies. (Figure from Rubin, Ford, and Thonnard, *Ap. J. Lett.*, *225*, L107, 1978.)

Since Galactic rotation depends on the distribution of mass, a great deal can be learned about the matter in galaxies by studying these curves. For instance, rigid-body rotation near the Galactic center implies that the mass must be roughly spherically distributed and the density nearly constant (see Problem 22.19). On the other hand, flat rotation curves suggest that the bulk of the mass in the outer portions of the Galaxy are spherically distributed with a density law that is proportional to $r^{-2}$. To see this, assume that $\Theta(r) = V$, where $V$ is a constant.[40] Then, from the equation for centripetal force and Newton's law of gravity, the force acting on a star of mass $m$ due to the mass $M_r$ of the Galaxy interior to the star's position at $r$ is[41]

$$\frac{mV^2}{r} = \frac{GM_r m}{r^2}.$$

Solving for $M_r$,

$$M_r = \frac{V^2 r}{G}, \tag{22.36}$$

and differentiating with respect to the radius of the distribution,

$$\frac{dM_r}{dr} = \frac{V^2}{G}.$$

[40]We are using $r$ for a spherically symmetric mass distribution here, rather than $R$ for cylindrical rotation in the Galactic plane. However, to obtain a rotation curve within the Galactic plane, we need only consider the special case of $r = R$.

[41]Recall that for spherically symmetric mass distributions, only the mass interior to $r$ will affect $m$; the contributions due to the mass exterior to $r$ will cancel.

If we now borrow the equation for mass conservation in a spherically symmetric system from stellar structure theory (Eq. 10.8),

$$\frac{dM_r}{dr} = 4\pi r^2 \rho,$$

we see that the mass density in the outer regions of the Galaxy must vary as

$$\rho(r) = \frac{V^2}{4\pi G r^2}. \tag{22.37}$$

This $r^{-2}$ density dependence is very different from the form determined by star counts in the portion of the Galaxy beyond the solar Galactocentric radius. Recall from Eq. (22.10) that the number density of stars in the luminous stellar halo is believed to vary as $r^{-3.5}$, a much more rapid drop-off than is evident from the flat rotation curve. It was this discrepancy that so surprised astronomers. As was mentioned at the end of Section 22.2, it appears that the majority of the mass in the Galaxy is in the form of nonluminous (dark) matter. Only through its gravitational influence on the luminous component of our Galaxy and satellite galaxies like the LMC and the SMC, and through gravitational lensing of light from background sources, does the dark matter make its presence known.

One modification that must be made to Eq. (22.37) is to force the density function to approach a constant value near the center so that it is consistent with the observational evidence of rigid-body rotation. As a result, we model the mass distribution of the dark matter halo in the form

$$\rho(r) = \frac{C_0}{(a^2 + r^2)}, \tag{22.38}$$

where $C_0 = 4.6 \times 10^8\ \mathrm{M}_\odot\ \mathrm{kpc}^{-1}$ and $a = 2.8$ kpc are chosen as parametric fits to the overall rotation curve. Note that for $r \gg a$, the $r^{-2}$ dependence is obtained, and $\rho \sim$ constant when $r \ll a$.

It is important to point out that Eq. (22.38) cannot be correct to arbitrarily large values of $r$. The reason for this is that the total amount of mass in the Galaxy would increase without bound since $M_r \propto r$. As a result, the density function for the dark matter halo must eventually terminate or at least decrease sufficiently rapidly that the mass integral $\int_0^\infty \rho(r)\, 4\pi r^2\, dr$ remains finite. In reality, other galaxies exist in our universe and their mass density functions may overlap our own. As a result, although galaxies appear to be separate luminous objects, their dark matter halos may actually merge in intergalactic space. Consequently, attempts to determination of the total mass of an individual galaxy could be an ill-posed problem.

22.19 (a) Show that rigid-body rotation near the Galactic center is consistent with a spherically symmetric mass distribution of constant density.

(b) Is the distribution of mass in the dark matter halo (Eq. 22.38) consistent with rigid-body rotation near the Galactic center? Why or why not?

22.20 Using the result of the "back-of-the-envelope" calculation for the density of dark matter (Eq. 22.37), estimate the mass density of dark matter in the solar neighborhood. Express your answer in units of g $\text{cm}^{-3}$, $\text{M}_\odot$ $\text{pc}^{-3}$, and $\text{M}_\odot$ $\text{AU}^{-3}$. How does your answer compare with the stellar mass density in the solar neighborhood?

22.21 (a) Assuming that Eq. (22.38) is valid for any arbitrary distance from the center of the Galaxy, show that the amount of dark matter interior to a radius is $r$ is given by the expression

$$M_r = 4\pi C_0 \left[ r - a \tan^{-1}\left(\frac{r}{a}\right)\right].$$

(b) If $5.5 \times 10^{11}$ $\text{M}_\odot$ of dark matter is located within 100 kpc of the Galactic center, determine $C_0$ in units of $\text{M}_\odot$ $\text{kpc}^{-1}$. Repeat your calculation if $1.3 \times 10^{12}$ $\text{M}_\odot$ is located within 230 kpc of the Galactic center.

(c) Estimate the amount of dark matter (in solar masses) within a radius of 50 kpc of the Galactic center. Compare your answer to the mass of the stellar halo.

22.22 (a) From the information given in Table 22.1 and in the text, determine the approximate mass-to-light ratio of the Galaxy interior to a radius of 25 kpc from the center.

(b) Repeat your calculation for a radius of 100 kpc. What can you conclude about the effect that dark matter might have on the average mass-to-light ratio of the universe?

22.23 The $r^{-2}$ dependence of Coulomb's electrostatic force law allows the construction of Gauss's law for electric fields, which has the form (in SI units)

$$\oint \mathbf{E} \cdot d\mathbf{A} = \frac{Q_{\text{in}}}{\epsilon_0},$$