Introduction:

The simple pendulum, with its regular clockwork motion is the epitome of deterministic dynamical systems. Newton's laws allow us to determine its period and predict its subsequent motion. Small variations in initial conditions result in only small variations in the motion. In this lab we will investigate how the introduction of non-linear forces into this system can lead to chaotic and unpredictable motion. In particular we will investigate under which conditions the pendulum system becomes sensitive to initial conditions.

Procedure:

Simple Pendulum

1. Set up a simple pendulum with a paper clip on the end of a thread, The thread should be about a metre long. Record the length.
2. Release the pendulum from rest and describe its subsequent motion.
   a) Does it return to its initial position?
   b) What happens to the motion over time?
   c) Does the period of the oscillations remain constant or change?
3. Measure the time for 10 oscillations of the pendulum for three different initial amplitudes. Does period depend on amplitude?
4. Shorten the pendulum and measure its length and period. Repeat this procedure for two new lengths. How does the period vary with length? Can you find a quantitative expression relating length to period?

Chaotic Pendulum

1. Set up the pendulum again so that the length is about a metre. Make sure the paperclip is suspended about one inch above the table surface or floor.
2. Mark off a piece of paper in square inch sections and tape it to the table surface so that the centre of the paper is directly below the paper clip.
3. Place two magnets 3 to 4 inches apart spaced equidistant from the centre of the paper aligned along the long axis of the paper.
4. Test the motion of the pendulum with the magnets in place. Make sure the magnets interact with the pendulum in a noticeable but not too extreme way. Describe the motion. Is the motion regular and predictable? Is there a well-defined period?
5. If you can find some release points for which the motion appears to be chaotic tape the magnets securely to the paper. Assign each magnet a unique label – for example you might want to use two different colours as labels since you will be using markers to colour your grid at the end of the lab.
6. Your task is to identify those release points for which the motion is predictable and those release points for which it is not predictable.
7. Release the pendulum from above one of the squares marked on your paper. The pendulum should eventually come to rest above one of the magnets. If the pendulum consistently comes to rest above the same magnet when it is released from above the same square mark that square with the label of that magnet. Otherwise mark the square as unpredictable.
8. Repeat this procedure for every square of the grid. You will eventually generate a pattern showing where the pendulum will end up when released. If you were to use a smaller and smaller grid your pattern would reveal increasing complexity on all scales. That is, it would be a fractal pattern. When you are finished colour in the grid

and compare your pattern with those of other members in the class. What are the similarities and differences?

Report

Your assignment is to write up your responses to the lab questions in the form of a short report, and submit it with the map you created showing eventual resting place of the pendulum when released. Your report should clearly indicate the data you recorded, with appropriate units. Your map should make use of color.

Equipment

50 rare earth magnets
meter stick
50 metal paper clips (ferromagnetic)
masking tape
25 stop watches
4 rolls of massless string – some kind of strong thread.
1 ream of 11X17 paper