

HOMEWORK WEEK 5

73. **SSM REASONING** The current  $I$  can be found by using Kirchoff's loop rule. Once the current is known, the voltage between points  $A$  and  $B$  can be determined.

**SOLUTION**

a. We assume that the current is directed clockwise around the circuit. Starting at the upper-left corner and going clockwise around the circuit, we set the potential drops equal to the potential rises:

$$\underbrace{(5.0 \Omega)I + (27 \Omega)I + 10.0 \text{ V} + (12 \Omega)I + (8.0 \Omega)I}_{\text{Potential drops}} = \underbrace{30.0 \text{ V}}_{\text{Potential rises}}$$

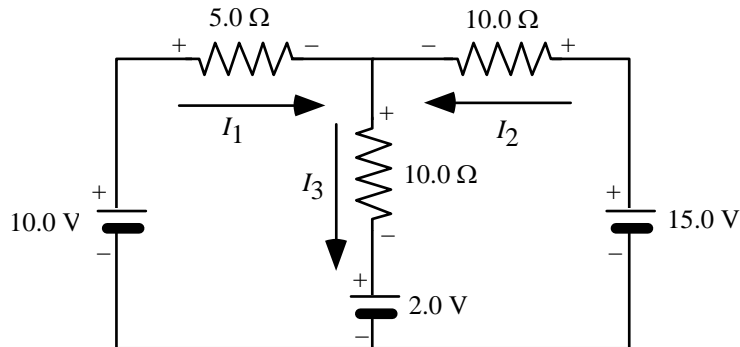
Solving for the current gives  $I = 0.38 \text{ A}$ .

b. The voltage between points  $A$  and  $B$  is

$$V_{AB} = 30.0 \text{ V} - (0.38 \text{ A})(27 \Omega) = 2.0 \times 10^1 \text{ V}$$

c. **Point B** is at the higher potential.

77. **SSM REASONING** We begin by labeling the currents in the three resistors. The drawing below shows the directions chosen for these currents. The directions are arbitrary, and if any of them is incorrect, then the analysis will show that the corresponding value for the current is negative.



We then mark the resistors with the plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (-). Thus, given the directions chosen for  $I_1$ ,  $I_2$ , and  $I_3$ , the plus and minus signs *must* be those shown in the drawing. We can now use Kirchoff's rules to find the voltage across the 5.0-Ω resistor.

**SOLUTION** Applying the loop rule to the left loop (and suppressing units for convenience) gives

$$5.0I_1 + 10.0I_3 + 2.0 = 10.0 \quad (1)$$

Similarly, for the right loop,

$$10.0I_2 + 10.0I_3 + 2.0 = 15.0 \quad (2)$$

If we apply the junction rule to the upper junction, we obtain

$$I_1 + I_2 = I_3 \quad (3)$$

Subtracting Equation (2) from Equation (1) gives

$$5.0I_1 - 10.0I_2 = -5.0 \quad (4)$$

We now multiply Equation (3) by 10 and add the result to Equation (2); the result is

$$10.0I_1 + 20.0I_2 = 13.0 \quad (5)$$

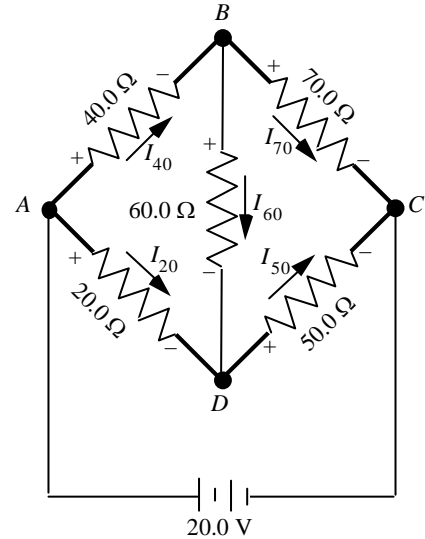
If we then multiply Equation (4) by 2 and add the result to Equation (5), we obtain  $20.0I_1 = 3.0$ , or solving for  $I_1$ , we obtain  $I_1 = 0.15$  A. The fact that  $I_1$  is positive means that the current in the drawing has the correct direction. The voltage across the 5.0- $\Omega$  resistor can be found from Ohm's law:

$$V = (0.15 \text{ A})(5.0 \Omega) = \boxed{0.75 \text{ V}}$$

Current flows from the higher potential to the lower potential, and the current through the 5.0- $\Omega$  flows from left to right, so the left end of the resistor is at the higher potential.

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79. **REASONING** To find the voltage between points  $B$  and  $D$ , we will find the current in the  $60.0\text{-}\Omega$  resistor and then use Ohm's law to find the voltage as  $V = IR$ . To find the current we will use Kirchoff's laws, the set-up for which is shown in the circuit diagram at the right. In this diagram we have marked the current in each resistor. It is  $I_{60}$  that we seek. Note that we have marked each resistor with plus and minus signs, to denote which end of the resistor is at the higher and which end is at the lower potential. Given our choice for the current directions, the plus and minus signs must be those shown, and they will help us apply Kirchoff's loop rule correctly. If our value for  $I_{60}$  turns out to be negative, it will mean that the actual direction for this current is opposite to that in the diagram.



**SOLUTION** Applying the junction rule to junctions  $B$  and  $D$  gives

$$\underbrace{I_{40} = I_{60} + I_{70}}_{\text{At junction } B} \quad (1)$$

$$\underbrace{I_{60} + I_{20} = I_{50}}_{\text{At junction } D} \quad (2)$$

Applying the loop rule to loops  $ABD$ ,  $BCD$ , and  $ADC$  (including the battery) gives

$$\underbrace{I_{40} (40.0 \Omega) + I_{60} (60.0 \Omega)}_{\text{Potential drops, loop } ABD} = \underbrace{I_{20} (20.0 \Omega)}_{\text{Potential rises, loop } ABD} \quad \text{or} \quad I_{40} (2.00) + I_{60} (3.00) = I_{20} \quad (3)$$

$$\underbrace{I_{70} (70.0 \Omega)}_{\text{Potential drops, loop } BCD} = \underbrace{I_{50} (50.0 \Omega) + I_{60} (60.0 \Omega)}_{\text{Potential rises, loop } BCD} \quad \text{or} \quad I_{70} (7.00) = I_{50} (5.00) + I_{60} (6.00) \quad (4)$$

$$\underbrace{I_{20} (20.0 \Omega) + I_{50} (50.0 \Omega)}_{\text{Potential drops, loop } ADC} = \underbrace{20.0 \text{ V}}_{\text{Potential rises, loop } ADC} \quad \text{or} \quad I_{20} (2.00) + I_{50} (5.00) = 2.00 \quad (5)$$

Equations (1)–(5) are five equations in five unknowns and must be solved simultaneously. Remember that it is  $I_{60}$  we seek, so our approach will be to eliminate the other four unknowns. Substituting  $I_{50}$  from Equation (2) into Equation (5) gives

$$I_{20} (7.00) + I_{60} (5.00) = 2.00 \quad (6)$$

Substituting  $I_{40}$  from Equation (1) into Equation (3) gives

$$I_{60} (5.00) + I_{70} (2.00) = I_{20} \quad (7)$$

Solving Equation (5) for  $I_{50}$  and substituting the result into Equation (4) gives

$$I_{70} (7.00) = 2.00 - I_{20} (2.00) + I_{60} (6.00) \quad (8)$$

Solving Equation (8) for  $I_{70}$  and substituting the result into Equation (7) gives

$$I_{60} (47.0) - I_{20} (11.0) = -4.00 \quad (9)$$

Solving Equation (6) for  $I_{20}$  and substituting the result into Equation (9) gives

$$I_{60} (47.0) - \left[ \frac{2.00 - I_{60} (5.00)}{7.00} \right] (11.0) = -4.00 \quad \text{or} \quad I_{60} = -0.0156 \text{ A}$$

Since this result is negative, the current in the  $60.0\text{-}\Omega$  resistor is opposite to that shown in the diagram in the reasoning step, that is, from point  $D$  up toward point  $B$ . Thus, point  $D$  is at a higher potential than point  $B$ , because conventional current is always directed from the high toward the low potential. Using Ohm's law, we find that the voltage between  $D$  and  $B$  is

$$V = I_{60} R = (0.0156 \text{ A})(60.0 \Omega) = \boxed{0.94 \text{ V, with point } D \text{ at the higher potential}}$$

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