Homework week 6

88. ***REASONING AND SOLUTION*** The equivalent capacitance of the circuit is

$$1/C = 1/(4.0\ \mu\text{F}) + 1/(6.0\ \mu\text{F}) + 1/(12.0\ \mu\text{F}) \qquad \text{or} \qquad C = 2.0\ \mu\text{F}$$

The total charge provided by the battery is, then,

$$Q = CV = (2.0 \times 10^{-6}\ \text{F})(50.0\ \text{V}) = 1.0 \times 10^{-4}\ \text{C}$$

This charge appears on each capacitor in a series circuit, so the voltage across the 4.0-μF capacitor is

$$V_1 = Q/C_1 = (1.0 \times 10^{-4}\ \text{C})/(4.0 \times 10^{-6}\ \text{F}) = \boxed{25\ \text{V}}$$

95. ***REASONING*** The time constant of an *RC* circuit is given by Equation 20.21 as $\tau = RC$, where *R* is the resistance and *C* is the capacitance in the circuit. The two resistors are wired in parallel, so we can obtain the equivalent resistance by using Equation 20.17. The two capacitors are also wired in parallel, and their equivalent capacitance is given by Equation 20.18. The time constant is the product of the equivalent resistance and equivalent capacitance.

SOLUTION The equivalent resistance of the two resistors in parallel is

$$\frac{1}{R_\text{P}} = \frac{1}{2.0\ \text{k}\Omega} + \frac{1}{4.0\ \text{k}\Omega} = \frac{3}{4.0\ \text{k}\Omega} \qquad \text{or} \qquad \text{R}_\text{P} = 1.3\ \text{k}\Omega \qquad (20.17)$$

The equivalent capacitance is

$$C_\text{P} = 3.0\ \mu\text{F} + 6.0\ \mu\text{F} = 9.0\ \mu\text{F} \qquad (20.18)$$

The time constant for the charge to build up is

$$\tau = R_\text{P}\, C_\text{P} = \left(1.3 \times 10^{3}\ \Omega\right)\left(9.0 \times 10^{-6}\ \text{F}\right) = \boxed{1.2 \times 10^{-2}\ \text{s}}$$

98. ***REASONING*** The charging of a capacitor is described by Equation 20.20, which provides a direct solution to this problem.

SOLUTION According to Equation 20.20, in a series RC circuit the charge *q* on the capacitor at a time *t* is given by

$$q = q_0\left(1 - e^{-t/\tau}\right)$$

where q_0 is the equilibrium charge that has accumulated on the capacitor after a very long time and τ is the time constant. For $q = 0.800 q_0$ this equation becomes

$$q = 0.800q_0 = q_0\left(1 - e^{-t/\tau}\right) \quad \text{or} \quad 0.200 = e^{-t/\tau}$$

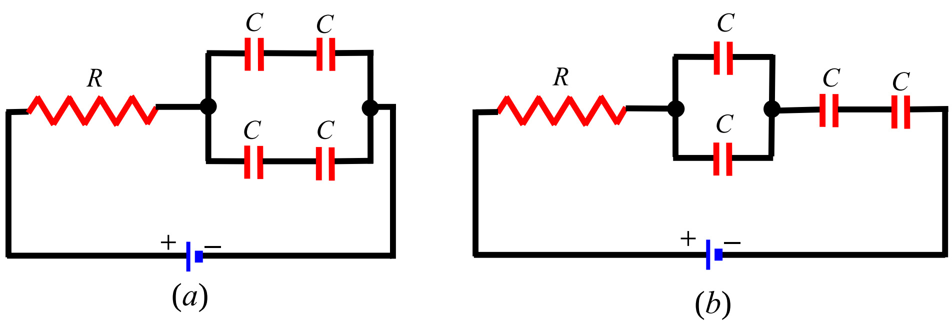
Taking the natural logarithm of both sides of this result gives

$$\ln(0.200) = \ln\left(e^{-t/\tau}\right) \quad \text{or} \quad \ln(0.200) = -\frac{t}{\tau}$$

Therefore, the number of time constants needed for the capacitor to be charged to 80.0% of its equilibrium charge is

$$\frac{t}{\tau} = -\ln(0.200) = -(-1.61) = \boxed{1.61}$$

99. ***REASONING*** In either part of the drawing the time constant τ of the circuit is $\tau = RC_{\text{eq}}$, according to Equation 20.21, where R is the resistance and C_{eq} is the equivalent capacitance of the capacitor combination. We will apply this equation to both circuits. To obtain the equivalent capacitance, we will analyze the capacitor combination in parts. For the parallel capacitors $C_{\text{P}} = C_1 + C_2 + C_3 + \ldots$ applies (Equation 20.18), while for the series capacitors $C_{\text{S}}^{-1} = C_1^{-1} + C_2^{-1} + C_3^{-1} + \ldots$ applies (Equation 20.19).



SOLUTION Using Equation 20.21, we write the time constant of each circuit as follows:

$$\tau_{\text{a}} = RC_{\text{eq, a}} \quad \text{and} \quad \tau_{\text{b}} = RC_{\text{eq, b}}$$

Dividing these two equations allows us to eliminate the unknown resistance algebraically:

$$\frac{\tau_{\text{b}}}{\tau_{\text{a}}} = \frac{RC_{\text{eq, b}}}{RC_{\text{eq, a}}} \quad \text{or} \quad \tau_{\text{b}} = \tau_{\text{a}}\left(\frac{C_{\text{eq, b}}}{C_{\text{eq, a}}}\right) \qquad (1)$$

To obtain the equivalent capacitance in part *a* of the drawing, we note that the two capacitors in series in each branch of the parallel combination have an equivalent capacitance C_S that can be determined using Equation 20.19

$$\frac{1}{C_S} = \frac{1}{C} + \frac{1}{C} \quad \text{or} \quad C_S = \tfrac{1}{2}C \qquad (2)$$

Using Equation 20.18, we find that the parallel combination in part *a* of the drawing has an equivalent capacitance of

$$C_{\text{eq, a}} = \tfrac{1}{2}C + \tfrac{1}{2}C = C \qquad (3)$$

To obtain the equivalent capacitance in part *b* of the drawing, we note that the two capacitors in series have an equivalent capacitance of $\tfrac{1}{2}C$, according to Equation (2). The two capacitors in parallel have an equivalent capacitance of $2C$, according to Equation 20.18. Finally, then, we have a series combination of $\tfrac{1}{2}C$ and $2C$, for which Equation 20.19 applies:

$$\frac{1}{C_{\text{eq, b}}} = \frac{1}{\tfrac{1}{2}C} + \frac{1}{2C} = \frac{5}{2C} \quad \text{or} \quad C_{\text{eq, b}} = \tfrac{2}{5}C \qquad (4)$$

Using Equations (3) and (4) in Equation (1), we find that

$$\tau_b = \tau_a \left(\frac{C_{\text{eq, b}}}{C_{\text{eq, a}}} \right) = (0.72 \text{ s}) \frac{\tfrac{2}{5}C}{C} = \boxed{0.29 \text{ s}}$$