Algebra self-assessment problems

Algebra Inventory

1. Simplify the following:

(a) $\frac{1}{a}(a^2+ba)$ $= a+b$

(b) $(a+b)^2-(a-b)^2$ $= a^2+2ab+b^2-(a^2-2ab+b^2)$
$= a^2+2ab+b^2-a^2+2ab-b^2 = 4ab$

(c) $\frac{x}{x+y}+\frac{y}{x-y}$ $= \frac{x(x-y)}{(x+y)(x-y)}+\frac{y(x+y)}{(x-y)(x+y)} = \frac{x^2-xy+xy+y^2}{x^2-xy+xy-y^2} = \frac{x^2+y^2}{x^2-y^2}$

(d) $\frac{a^2}{b}\div\frac{2}{ba}$ $= \frac{a^2}{b}\times\frac{ba}{2} = \frac{a^3}{2}$

2. Solve for x

(a) $\frac{1}{3}-\frac{1}{5}x=2$

$\frac{1}{3}=\frac{1}{5}x+2$

$\frac{1}{3}-2=\frac{x}{5}$

$-\frac{5}{3}=\frac{x}{5}$

$x=-\frac{25}{3}$

(b) $\frac{x-1}{2}=\frac{3}{x+2}$

$(x-1)(x+2)=6$

$x^2-x+2x-2=6$

$x^2+x-8=0$

Using the quadratic formula:

$x=\frac{-1\pm\sqrt{1+32}}{2}=\frac{-1+\sqrt{33}}{2}$ or $\frac{-1-\sqrt{33}}{2}$

(c) $2^x\times 2^{2x}\times 4=8$

$2^{x+2x}\times 2^2=2^3$

$2^{3x+2}=2^3$

$3x+2=3$

$3x=1$

$x=\frac{1}{3}$

(d) $3\log x-1=5$

$3\log_{10}x=6$

$\log_{10}x=2$

$10^2=x$

$x=100$

3. In each of the following equations solve for y as a function of x

(a) $3x + 2y = 1$

$2y = 1 - 3x$

$y = \frac{1}{2} - \frac{3x}{2}$

(b) $x = \sqrt{y^2 - 4}$

$x^2 = y^2 - 4 \quad (\text{and } y^2 - 4 \geq 0)$

$y^2 = x^2 + 4$

$y = \pm\sqrt{x^2 + 4} \quad (x \geq 0)$

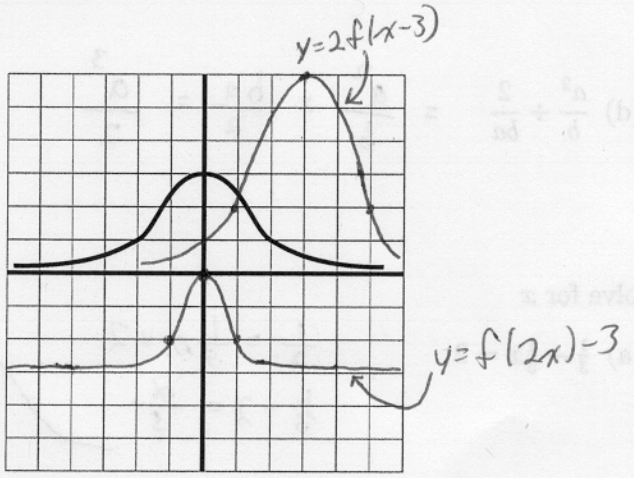
(c) $x = 10^{2y} - 4$

$x + 4 = 10^{2y}$

$\log_{10}(x+4) = 2y$

$y = \frac{1}{2}\log_{10}(x+4)$

4. The curve $y = f(x)$ is sketched below. Sketch $y = 2f(x-3)$ and $y = f(2x) - 3$ on the same graph, clearly indicating which is which.



5. Simplify the following trigonometric expressions

(a) $\sin^2 x + \cos^2 x = 1$

(b) $\dfrac{\sin x}{\cos x} = \tan x$

(c) $\cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$