Week 1 Assignment

Mathematical Models in Biology ***An Introduction***
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Ch 1.1 - Ex(2,7,10)
Ch 1.2 - Ex(2,5,7,8,9,11)

1.1.2

$$
\begin{aligned}
f_{t+1} &= 2f_t \;\; f_0 = 1 \\
f_t &= f_0 2^{2t} \text{ where } t \text{ is number of hours}
\end{aligned}
$$

t	0	1	2	3	4	5	6	7	8	9	10
f	1	4	16	64	256	1024	4096	16384	65536	262144	1048576

The main question that arises is how does cell replication rate change from doubling every half hour initially to yield 30,000 cells after 10 hours?

1.1.7 a) The models $P_t = kP_{t-1}$ and $\Delta P = rP$ represents growing populations when k is any number in the range **k** $>$ **1** and when r is any number in the range **r** $>$ **0**.

b) The models $P_t = kP_{t-1}$ and $\Delta P = rP$ represents declining populations when k is any number in the range **k** $<$ **1** and when r is any number in the range **-1** $\leq$ **r** $<$ **0**.

c) The models $P_t = kP_{t-1}$ and $\Delta P = rP$ represents stable populations when k is any number in the range **k** $=$ **1** and when r is **r** $=$ **0**.

1.1.10 a) . . . . $\Delta P = 0$

b) . . . . no change is happening at that point.

c) Yes the model $P_{t+1} = (1 + r)P_t$ has steady states. If $P_0 = 0$ then $P_{t+1} = 0$ making the $\Delta P = 0$. And if $r = 0$ than $P_t = P_{t+1}$ which also gives a $\Delta P = 0$.

1.2.2 For the model $\Delta P = 1.3P(1 - P/10)$, $P > 10$ will be negative, $P < 10$ will be positive, which can be seen from the $P/10$ term. Biologically this is your carrying capacity.

1.2.5 a) Using $\Delta P = P_{t+1} - P_t$

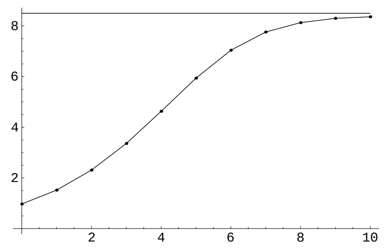
$$
\begin{aligned}
P_{t+1} &= P_t + .2P(10 - P_t) \;\Rightarrow\; \Delta P = .2P(10 - P) \\
\Delta P &= .2P(10 - P) = sP(k - P) \\
\Delta P &= .2P(10 - P) = 2P(1 - \frac{P}{10}) = rP(1 - \frac{P}{k}) \\
\Delta P &= 2P(1 - \frac{P}{10}) = 2P - .2P^2 = tP - uP^2 \\
P_{t+1} &= P_t + .2P(10 - P_t) = 3P - .2P^2 = cP - wP^2
\end{aligned}
$$

b)

$$
\begin{aligned}
P_{t+1} &= 2.5P_t - .2P_t^2 = vP_t - wP_t^2 \\
\Delta P &= 1.5P_t - .2P_t^2 = tP - uP^2 \\
\Delta P &= .2P(7.5 - P) = sP(k - P) \\
\Delta P &= 1.5P(1 - \frac{P}{7.5}) = rP(1 - \frac{P}{k})
\end{aligned}
$$

1.2.7 If we graph the points than we can easily see that the information does fit with the logistic equation. To set up the logistic equation for this graph we need the growth factor and the carrying capacity. By looking at the graph we see that the carnying capacity is between 8.5 and 9, which is the upper limit where the curve begins to level off. Then to find the growth factor we just take $\Delta P = (P_{t+1} - P_t)$ and divide it by P_t doing this for the first P_t yields .567. So our logistic equation would be

$$\Delta P = .567P(1 - \frac{P}{8.5})$$



(a) P vs t

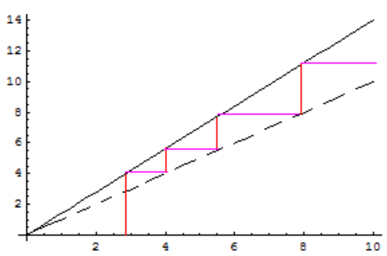
1.2.8 a)

$$\begin{aligned}
N_{t+1} &= N_t + .2N_t(1 - \frac{N}{200000}) \\
N_t &= M_t * 1000 \\
1000M_{t+1} &= 1000M_t + 200M_t(1 - \frac{1000M_t}{200000}) \\
M_{t+1} &= M_t + .2M_t(1 - \frac{M_t}{200})
\end{aligned}$$

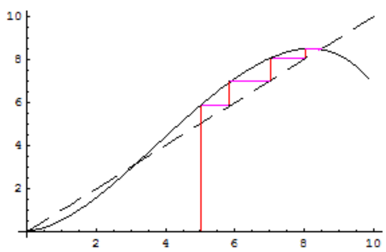
b)

$$N_{t+1} = N_t + .2N_t(1 - N_t)$$

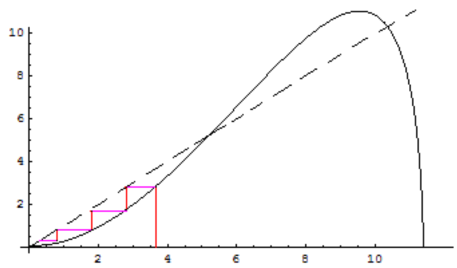
1.2.9



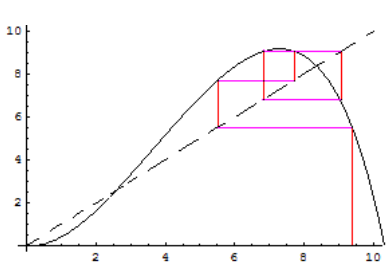
(b) $P_{t+1} vs. P_t$



(c) $P_{t+1} vs. P_t$

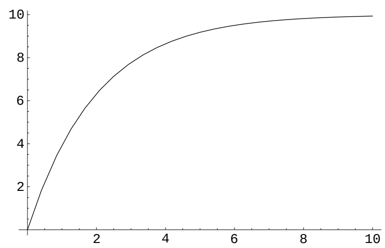


(d) $P_{t+1} vs. P_t$



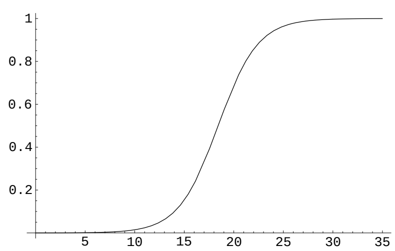
(e) $P_{t+1} vs. P_t$

1.2.11 a) The reason $\Delta N = r(K - N)$ is the model is that ΔN is the change in the value of N separated from K, so you can't have more N than K, hence $(K - N)$, and r is the rate of chemical reaction. Values for r should be $0 < r < 1$ because $r < 0$ would represent an impossible reaction chemical reaction of N going increasingly negative to infinity and never being stopped by a limititing prosses once the system was run. This can't happen in reality.Also $r > 1$ would allow you to have $\Delta N > K$, if $N_0 = 0$ the first time step would yield $N > K$ which can't happen because N is determined from K. $N_0 = 0$ because N comes from K.



(f) N(t) vs. t

b) $\Delta N = rN(K - N)$ is a model for an autocatalitic reaction because ΔN depends on the amount of N present. If $N_0 = 0$ no reaction hapends, but when $N > 0$ than a reaction starts taking place. The graph would start out with a small positive slope when N is much less that K and then it is going to begin to increase its slope. At some point the slop would reach a maximum slope and then begin to tale off again once N started approaching K.



(g) N(t) vs. t