This is a assessment assignment. This means I will collect and mark the work you do. You may refer to notes and your textbook. You may also work with other students. If you do work with other students please indicate who you worked with, and on which questions you worked together.

1. Suppose that the frequencies of two alleles A and a in a population is $p(A) = 0.3$ and $p(a) = 0.7$

   (a) What fraction of the population is expected to be heterozygotes?

   (b) What is the probability of finding exactly 4 heterozygotes in a group of 6 individuals?

2. In a large population there are two alleles for a particular gene, A and a, which occur with relative frequency p and q respectively (with $q = 1 - p$). The relative fitness of the genotypes are $W_{AA} = 1 - s$, $W_{Aa} = 1$, and $W_{aa} = 1 - t$.

   (a) Show that under random mating the difference equation describing how p changes is given by

   $$p_{t+1} = \frac{p_t(1 - sp_t)}{1 - sp_t^2 - t(1 - p_t)^2}$$

   (b) Hence, show that the allele frequencies reach an equilibrium when $p = \dfrac{t}{t+s}$.

   (c) Now assume that population is initially made up almost entirely of homozygote aa individuals, but that there is small proportion of mutant A alleles (so that $p_0 = P(A) \ll 1$ and $q_0 = P(a) \approx 1$, show that the initial growth in the frequency of A is approximately exponential and determine the growth rate.

3. Suppose two species A and B occur in the population with relative frequency x and y, and with fitness a and b respectively, so that the equations 2.13 hold

   $$\begin{aligned} \dot{x} &= x(a - \phi) \\ \dot{y} &= y(b - \phi) \end{aligned}$$

   where ϕ is a "selection" function, to be determined, that serves to maintain $x + y = 1$.

   (a) Prove that in order to maintain $x+y = 1$ the function ϕ must take the form $\phi = ax+by$. (Hint: Try adding the two equations above and use the fact that if $x + y = 1$ then $\dot{x} + \dot{y} = 0$).

   (b) Hence show that the above system reduces to the single equation

   $$\dot{x} = (a - b)x(1 - x)$$

4. The above equations represent both reproduction (a and b) and selection ϕ. Now suppose that it is also possible for A to mutate to B with probability u and B mutates to A with probability v modify the equations in the previous question to account for reproduction, selection and mutation.

5. In the Hawk-Dove game two individuals compete for a resource. An individual using the Hawk strategy postures first but escalates to a fight if threatened. An individual who using the Dove strategy postures first, but retreats if threatened. There is a benefit b to the individual who gains a resource and a cost c to individuals who are injured in a fight over the resource. What are the values of b and c corresponding to the Payoff matrix below

	H	D
H	-2	2
D	0	1

What are the values of b and c in the above matrix?

6. There is an assumption in the above model that there is no cost to a dove in competing for a resource. Now assume that a dove must expend some amount of energy e in posturing over the resource, but still backs down when its opponent escalates.

   (a) Write down the new payoff matrix for the Hawk-Dove game

   (b) Given that it probably costs more to fight than to posture, state an upper bound on e so that this game makes sense on these grounds.

   (c) Find, in terms of e, the frequency of Hawks x when Hawks and Doves coexist in a stable equilibrium for this new game.

   (d) State a lower bound on e so that the pure Hawk strategy is an ESS.