Homework week 3

2. ***REASONING*** The law of conservation of electric charges states that the net electric charge of an isolated system remains constant. Initially the plate-rod system has a net charge of $-3.0\ \mu C + 2.0\ \mu C = -1.0\ \mu C$. After the transfer this charge is shared equally by both objects, so that each carries a charge of $-0.50\ \mu C$. Therefore, $2.5\ \mu C$ of negative charge must be transferred from the plate to the rod. To determine how many electrons this is, we will divide this charge magnitude by the magnitude of the charge on a single electron.

SOLUTION The magnitude of the charge on an electron is e, so that the number N of electrons transferred is

$$N = \frac{\text{Magnitude of transferred charge}}{e} = \frac{2.5\times10^{-6}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.6\times10^{13}}$$

5. **SSM** ***REASONING*** Identical conducting spheres equalize their charge upon touching. When spheres A and B touch, an amount of charge $+q$, flows from A and instantaneously neutralizes the $-q$ charge on B leaving B momentarily neutral. Then, the remaining amount of charge, equal to $+4q$, is equally split between A and B, leaving A and B each with equal amounts of charge $+2q$. Sphere C is initially neutral, so when A and C touch, the $+2q$ on A splits equally to give $+q$ on A and $+q$ on C. When B and C touch, the $+2q$ on B and the $+q$ on C combine to give a total charge of $+3q$, which is then equally divided between the spheres B and C; thus, B and C are each left with an amount of charge $+1.5q$.

SOLUTION Taking note of the initial values given in the problem statement, and summarizing the final results determined in the *Reasoning* above, we conclude the following:

a. Sphere C ends up with an amount of charge equal to $\boxed{+1.5q}$.

b. The charges on the three spheres before they were touched, are, according to the problem statement, $+5q$ on sphere A, $-q$ on sphere B, and zero charge on sphere C. Thus, the total charge on the spheres is $+5q - q + 0 = \boxed{+4q}$.

c. The charges on the spheres after they are touched are $+q$ on sphere A, $+1.5q$ on sphere B, and $+1.5q$ on sphere C. Thus, the total charge on the spheres is $+q + 1.5q + 1.5q = \boxed{+4q}$.

9. SSM WWW ***REASONING*** Initially, the two spheres are neutral. Since negative charge is removed from the sphere which loses electrons, it then carries a net positive charge. Furthermore, the neutral sphere to which the electrons are added is then negatively charged. Once the charge is transferred, there exists an electrostatic force on each of the two spheres, the magnitude of which is given by Coulomb's law (Equation 18.1), $F = k|q_1||q_2|/r^2$.

SOLUTION

a. Since each electron carries a charge of -1.60×10^{-19} C, the amount of negative charge removed from the first sphere is

$$\left(3.0\times10^{13}\text{ electrons}\right)\left(\frac{1.60\times10^{-19}\text{ C}}{1\text{ electron}}\right) = 4.8\times10^{-6}\text{ C}$$

Thus, the first sphere carries a charge $+4.8 \times 10^{-6}$ C, while the second sphere carries a charge -4.8×10^{-6} C. The magnitude of the electrostatic force that acts on each sphere is, therefore,

$$F = \frac{k|q_1||q_2|}{r^2} = \frac{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(4.8\times10^{-6}\text{ C}\right)^2}{\left(0.50\text{ m}\right)^2} = \boxed{0.83\text{ N}}$$

b. Since the spheres carry charges of opposite sign, the force is $\boxed{\text{attractive}}$.

11. **SSM** ***REASONING AND SOLUTION*** The net electrostatic force on charge 3 at $x = +3.0$ m is the vector sum of the forces on charge 3 due to the other two charges, 1 and 2. According to Coulomb's law (Equation 18.1), the magnitude of the force on charge 3 due to charge 1 is

$$F_{13} = \frac{k|q_1||q_3|}{r_{13}^2}$$

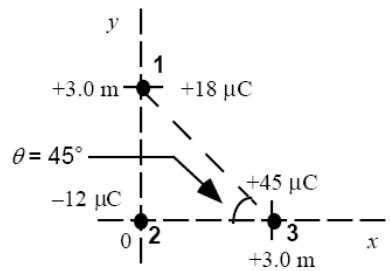


Figure 1

where the distance between charges 1 and 3 is r_{13}.

According to the Pythagorean theorem, $r_{13}^2 = x^2 + y^2$. Therefore,

$$F_{13} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(18\times10^{-6}\ \text{C}\right)\left(45\times10^{-6}\ \text{C}\right)}{(3.0\ \text{m})^2 + (3.0\ \text{m})^2} = 0.405\ \text{N}$$

Charges 1 and 3 are equidistant from the origin, so that $\theta = 45°$ (see Figure 1). Since charges 1 and 3 are both positive, the force on charge 3 due to charge 1 is repulsive and along the line that connects them, as shown in Figure 2. The components of F_{13} are:

$$F_{13x} = F_{13}\cos 45° = 0.286\ \text{N} \qquad \text{and} \qquad F_{13y} = -F_{13}\sin 45° = -0.286\ \text{N}$$

The second force on charge 3 is the attractive force (opposite signs) due to its interaction with charge 2 located at the origin. The magnitude of the force on charge 3 due to charge 2 is, according to Coulomb's law ,

$$F_{23} = \frac{k|q_2||q_3|}{r_{23}^2} = \frac{k|q_2||q_3|}{x^2}$$

$$= \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(12\times10^{-6}\ \text{C}\right)\left(45\times10^{-6}\ \text{C}\right)}{(3.0\ \text{m})^2}$$

$$= 0.539\ \text{N}$$

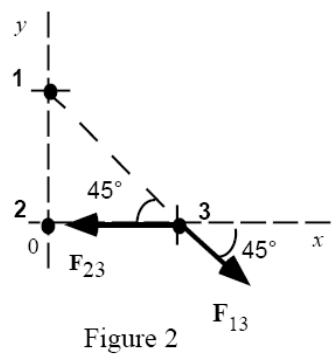


Figure 2

Since charges 2 and 3 have opposite signs, they attract each other, and charge 3 experiences a force to the left as shown in Figure 2. Taking up and to the right as the positive directions, we have

$$F_{3x} = F_{13x} + F_{23x} = +0.286\text{ N} - 0.539\text{ N} = -0.253\text{ N}$$

$$F_{3y} = F_{13y} = -0.286\text{ N}$$

Using the Pythagorean theorem, we find the magnitude of F_3 to be

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-0.253\text{ N})^2 + (-0.286\text{ N})^2} = \boxed{0.38\text{ N}}$$

The direction of F_3 relative to the $-x$ axis is specified by the angle ϕ, where

$$\phi = \tan^{-1}\left(\frac{0.286\text{ N}}{0.253\text{ N}}\right) = \boxed{49^\circ \text{ below the } -x \text{ axis}}$$

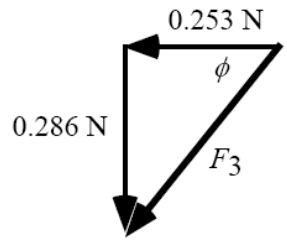
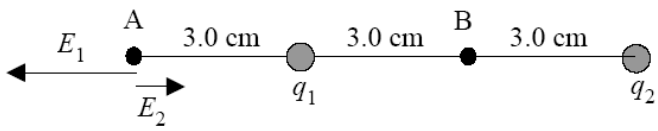


Figure 3

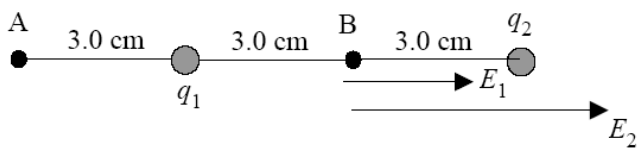
31. SSM ***REASONING***

a. The drawing shows the two point charges q_1 and q_2. Point A is located at $x = 0$ cm, and point B is at $x = +6.0$ cm.



Since q_1 is positive, the electric field points away from it. At point A, the electric field E_1 points to the left, in the $-x$ direction. Since q_2 is negative, the electric field points toward it. At point A, the electric field E_2 points to the right, in the $+x$ direction. The net electric field is $E = -E_1 + E_2$. We can use Equation 18.3, $E = k|q|/r^2$, to find the magnitude of the electric field due to each point charge.

b. The drawing shows the electric field produced by the charges q_1 and q_2 at point B, which is located at $x = +6.0$ cm.



Since q_1 is positive, the electric field points away from it. At point B, the electric field points to the right, in the $+x$ direction. Since q_2 is negative, the electric field points toward it. At point B, the electric field points to the right, in the $+x$ direction. The net electric field is $E = +E_1 + E_2$.

SOLUTION

a. The net electric field at the origin (point A) is $E = -E_1 + E_2$:

$$E = -E_1 + E_2 = \frac{-k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= \frac{-\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.5\times10^{-6}\ \text{C}\right)}{\left(3.0\times10^{-2}\ \text{m}\right)^2} + \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(21\times10^{-6}\ \text{C}\right)}{\left(9.0\times10^{-2}\ \text{m}\right)^2}$$

$$= \boxed{-6.2\times10^7\ \text{N/C}}$$

The minus sign tells us that the net electric field points along the $-x$ axis.

b. The net electric field at $x = +6.0$ cm (point B) is $E = E_1 + E_2$:

$$E = E_1 + E_2 = \frac{k|q_1|}{r_1^2} + \frac{k|q_2|}{r_2^2}$$

$$= \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(8.5\times10^{-6}\ \mathrm{C}\right)}{\left(3.0\times10^{-2}\ \mathrm{m}\right)^2} + \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(21\times10^{-6}\ \mathrm{C}\right)}{\left(3.0\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{+2.9\times10^8\ \mathrm{N/C}}$$

The plus sign tells us that the net electric field points along the $+x$ axis.