Workshop week 3

7. **SSM** **REASONING AND SOLUTION** The magnitude of the force of attraction between
    the charges is given by Coulomb's law (Equation 18.1): $F = k|q_1||q_2|/r^2$, where
    $|q_1|$ and $|q_2|$ are the magnitudes of the charges and r is the separation of the charges. Let
    F_A and F_B represent the magnitudes of the forces between the charges when the separations
    are r_A and $r_B = r_A/9$, respectively. Then

    $$\frac{F_B}{F_A} = \frac{k|q_1||q_2|/r_B^2}{k|q_1||q_2|/r_A^2} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{r_A}{r_A/9}\right)^2 = (9)^2 = 81$$

    Therefore, we can conclude that $F_B = 81F_A = (81)(1.5 \text{ N}) = \boxed{120 \text{ N}}$.

14. **REASONING**
    a. The magnitude of the electrostatic force that acts on each sphere is given by Coulomb's
    law as $F = k|q_1||q_2|/r^2$, where $|q_1|$ and $|q_2|$ are the magnitudes of the charges, and r is the
    distance between the centers of the spheres.

    b. When the spheres are brought into contact, the net charge after contact and separation
    must be equal to the net charge before contact. Since the spheres are identical, the charge
    on each after being separated is one-half the net charge. Coulomb's law can be applied
    again to determine the magnitude of the electrostatic force that each sphere experiences.

    SOLUTION
    a. The magnitude of the force that each sphere experiences is given by Coulomb's law as:

    $$F = \frac{k|q_1||q_2|}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(20.0 \times 10^{-6} \text{ C}\right)\left(50.0 \times 10^{-6} \text{ C}\right)}{\left(2.50 \times 10^{-2} \text{ m}\right)^2} = \boxed{1.44 \times 10^4 \text{ N}}$$

    Because the charges have opposite signs, the force is $\boxed{\text{attractive}}$.

    b. The net charge on the spheres is $-20.0 \ \mu\text{C} + 50.0 \ \mu\text{C} = +30.0 \ \mu\text{C}$. When the spheres are
    brought into contact, the net charge after contact and separation must be equal to the net
    charge before contact, or $+30.0 \ \mu\text{C}$. Since the spheres are identical, the charge on each
    after being separated is one-half the net charge, so $q_1 = q_2 = +15.0 \ \mu\text{C}$. The electrostatic
    force that acts on each sphere is now

    $$F = \frac{k|q_1||q_2|}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(15.0 \times 10^{-6} \text{ C}\right)\left(15.0 \times 10^{-6} \text{ C}\right)}{\left(2.50 \times 10^{-2} \text{ m}\right)^2} = \boxed{3.24 \times 10^3 \text{ N}}$$

    Since the charges now have the same signs, the force is $\boxed{\text{repulsive}}$.

17. **REASONING**

a. There are two electrostatic forces that act on q_1; that due to q_2 and that due to q_3. The magnitudes of these forces can be found by using Coulomb's law. The magnitude and direction of the net force that acts on q_1 can be determined by using the method of vector components.

b. According to Newton's second law, Equation 4.2b, the acceleration of q_1 is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force.
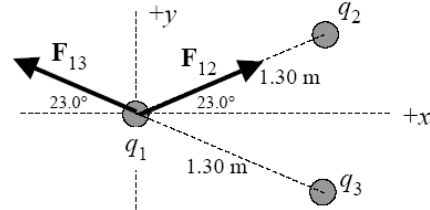
SOLUTION

a. The magnitude F_{12} of the force exerted on q_1 by q_2 is given by Coulomb's law, Equation 18.1, where the distance is specified in the drawing:

$$F_{12} = \frac{k|q_1||q_2|}{r_{12}^2} = \frac{\left(8.99 \times 10^9 \ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00 \times 10^{-6} \ \text{C}\right)\left(5.00 \times 10^{-6} \ \text{C}\right)}{(1.30 \ \text{m})^2} = 0.213 \ \text{N}$$

Since the magnitudes of the charges and the distances are the same, the magnitude of $\mathbf{F}_{13}$ is the same as the magnitude of $\mathbf{F}_{12}$, or $F_{13} = 0.213$ N. From the drawing it can be seen that the x-components of the two forces cancel, so we need only to calculate the y components of the forces.

Force	y component
$\mathbf{F}_{12}$	$+F_{12} \sin 23.0° = +(0.213 \ \text{N}) \sin 23.0° = +0.0832 \ \text{N}$
$\mathbf{F}_{13}$	$+F_{13} \sin 23.0° = +(0.213 \ \text{N}) \sin 23.0° = +0.0832 \ \text{N}$
$\mathbf{F}$	$F_y = +0.166 \ \text{N}$

Thus, the net force is $\boxed{\mathbf{F} = +0.166 \ \text{N} \ (\text{directed along the } +y \text{ axis})}$.

b. According to Newton's second law, Equation 4.2b, the acceleration of q_1 is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force:

$$a = \frac{F}{m} = \frac{+0.166 \ \text{N}}{1.50 \times 10^{-3} \ \text{kg}} = \boxed{+111 \ \text{m/s}^2}$$

where the plus sign indicates that $\boxed{\text{the acceleration is along the } +y \text{ axis}}$.

23. $\boxed{\text{SSM}}$ *REASONING* The charged insulator experiences an electric force due to the presence of the charged sphere shown in the drawing in the text. The forces acting on the insulator are the downward force of gravity (i.e., its weight, $W = mg$), the electrostatic force $F = k|q_1||q_2|/r^2$ (see Coulomb's law, Equation 18.1) pulling to the right, and the tension T in the wire pulling up and to the left at an angle θ with respect to the vertical as shown in the drawing in the problem statement. We can analyze the forces to determine the desired quantities θ and T.

SOLUTION.
a. We can see from the diagram given with the problem statement that

$$T_x = F \qquad \text{which gives} \qquad T\sin\theta = k|q_1||q_2|/r^2$$

and

$$T_y = W \qquad \text{which gives} \qquad T\cos\theta = mg$$

Dividing the first equation by the second yields

$$\frac{T\sin\theta}{T\cos\theta} = \tan\theta = \frac{k|q_1||q_2|/r^2}{mg}$$

Solving for θ, we find that

$$\theta = \tan^{-1}\left(\frac{k|q_1||q_2|}{mgr^2}\right)$$

$$= \tan^{-1}\left[\frac{(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(0.600\times10^{-6} \text{ C})(0.900\times10^{-6} \text{ C})}{(8.00\times10^{-2} \text{ kg})(9.80 \text{ m/s}^2)(0.150 \text{ m})^2}\right] = \boxed{15.4°}$$

b. Since $T\cos\theta = mg$, the tension can be obtained as follows:

$$T = \frac{mg}{\cos\theta} = \frac{(8.00\times10^{-2} \text{ kg})(9.80 \text{ m/s}^2)}{\cos 15.4°} = \boxed{0.813 \text{ N}}$$