Workshop week 5

Conceptual questions

3. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** Two materials have different resistivities. Two wires of the same length are made, one from each of the materials. The resistance of each wire is given by Equation 20.3: $R = \rho L / A$, where ρ is the resistivity of the wire material, and L and A are, respectively, the length and cross-sectional area of the wire. Even when the wires have the same length, they may have the same resistance, if the cross-sectional areas of the wires are chosen so that the ratio $\rho L / A$ is the same for each.

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4. ***REASONING AND SOLUTION*** The resistance of a wire is given by Equation 20.3: $R = \rho L / A$, where ρ is the resistivity of the wire material, L is the length of the wire, and A is its cross-sectional area. Since the cross-sectional area is proportional to the square of the diameter, a doubling of the diameter causes the cross-sectional area to be increased four-fold. From Equation 20.3, we see that doubling both the diameter and length causes the resistance of the wire to be reduced by a factor of 2.

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5. ***REASONING AND SOLUTION*** One electrical appliance operates with a voltage of 120 V, while another operates with a voltage of 240 V. The power used by either appliance is given by Equation 20.6c: $P = V^2 / R$. Without knowing the resistance R of each appliance, no conclusion can be reached as to which appliance, if either, uses more power.

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6. ***REASONING AND SOLUTION*** Two light bulbs are designed for use at 120 V and are rated at 75 W and 150 W. The power used by either bulb is given by Equation 20.6c: $P = V^2 / R$. We see from Equation 20.6c that, at constant voltage, the power used by a bulb is inversely proportional to the resistance of the filament. Therefore, the filament resistance is greater for the 75-W bulb.

Problems

10. ***REASONING*** The resistance R of a wire that has a length L and a cross-sectional area A is given by Equation 20.3 as $R = \rho \dfrac{L}{A}$. Both wires have the same length and cross-sectional area. Only the resistivity ρ of the wire differs, and Table 20.1 gives the following values: $\rho_{\text{Aluminum}} = 2.82 \times 10^{-8}\ \Omega\cdot\text{m}$ and $\rho_{\text{Copper}} = 1.72 \times 10^{-8}\ \Omega\cdot\text{m}$. Applying Equation 20.3 to both wires and dividing the two equations will allow us to eliminate the unknown length and cross-sectional area algebraically and solve for the resistance of the copper wire.

SOLUTION Applying Equation 20.3 to both wires gives

$$R_{\text{Copper}} = \rho_{\text{Copper}} \frac{L}{A} \quad \text{and} \quad R_{\text{Aluminum}} = \rho_{\text{Aluminum}} \frac{L}{A}$$

Dividing these two equations, eliminating L and A algebraically, and solving the result for R_{Copper} give

$$\frac{R_{\text{Copper}}}{R_{\text{Aluminum}}} = \frac{\rho_{\text{Copper}} L/A}{\rho_{\text{Aluminum}} L/A} = \frac{\rho_{\text{Copper}}}{\rho_{\text{Aluminum}}}$$

$$R_{\text{Copper}} = R_{\text{Aluminum}}\left(\frac{\rho_{\text{Copper}}}{\rho_{\text{Aluminum}}}\right) = (0.20\ \Omega)\left(\frac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{2.82\times10^{-8}\ \Omega\cdot\text{m}}\right) = \boxed{0.12\ \Omega}$$

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11. ***REASONING AND SOLUTION*** The resistance of the cable is

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

Since $A = \pi r^2$, the radius of the cable is

$$r = \sqrt{\frac{\rho L I}{\pi V}} = \sqrt{\frac{\left(1.72\times10^{-8}\ \Omega\cdot\text{m}\right)(0.24\ \text{m})(1200\ \text{A})}{\pi\left(1.6\times10^{-2}\ \text{V}\right)}} = \boxed{9.9\times10^{-3}\ \text{m}}$$

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12. ***REASONING AND SOLUTION*** Using Equation 20.3 and the resistivity of aluminum from Table 20.1, we find

$$R = \frac{\rho L}{A} = \frac{\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)\left(10.0\times10^{3}\ \text{m}\right)}{4.9\times10^{-4}\ \text{m}^2} = \boxed{0.58\ \Omega}$$

18. ***REASONING*** The length L of the wire is related to its resistance R and cross-sectional area A by $L = AR/\rho$ (see Equation 20.3), where ρ is the resistivity of tungsten. The resistivity is known (see Table 20.1), and the cross-sectional area can be determined since the radius of the wire is given. The resistance can be obtained from Ohm's law as the voltage divided by the current.

SOLUTION The length L of the wire is

$$L = \frac{AR}{\rho} \tag{1}$$

Since the cross-section of the wire is circular, its area is $A = \pi r^2$, where r is the radius of the wire. According to Ohm's law (Equation 20.2), the resistance R is related to the voltage

V and current I by $R = V/I$. Substituting the expressions for A and R into Equation (1) gives

$$L = \frac{AR}{\rho} = \frac{\left(\pi r^2\right)\left(\frac{V}{I}\right)}{\rho} = \frac{\left[\pi\left(0.0030\times10^{-3}\ \text{m}\right)^2\right]\left(\frac{120\ \text{V}}{1.24\ \text{A}}\right)}{5.6\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{0.049\ \text{m}}$$

21. ***REASONING AND SOLUTION*** According to Equation 20.6c, the power delivered to the iron is

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{24\ \Omega} = \boxed{6.0 \times 10^2\ \text{W}}$$

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22. ***REASONING*** To find the current, we can use the relation that the power P is the product of the current I and the voltage V, since the power and voltage are known.

SOLUTION Solving $P = IV$ (Equation 20.6a) for the current, we have

$$I = \frac{P}{V} = \frac{0.11\ \text{W}}{4.5\ \text{V}} = \boxed{0.024\ \text{A}}$$

26. ***REASONING AND SOLUTION*** We know that the resistance of the wire can be obtained from

$$P = V^2/R \qquad \text{or} \qquad R = V^2/P$$

We also know that $R = \rho L/A$. Solving for the length, noting that $A = \pi r^2$, and using $\rho = 100 \times 10^{-8}\ \Omega\cdot\text{m}$ from Table 20.1, we find

$$L = \frac{RA}{\rho} = \frac{\left(V^2/P\right)\left(\pi r^2\right)}{\rho} = \frac{V^2 \pi r^2}{\rho P} = \frac{(120\ \text{V})^2\, \pi \left(6.5 \times 10^{-4}\ \text{m}\right)^2}{\left(100 \times 10^{-8}\ \Omega\cdot\text{m}\right)\left(4.00 \times 10^2\ \text{W}\right)} = \boxed{50\ \text{m}}$$