1. Find the equation of the curve $y(x)$ passing through the points $(0,0)$ and $(1,0)$ for which the integral expression $V[y] = \int_0^1 (y')^2 dx$ takes a minimum value, given the constraint $\int_0^1 y^2 dx = 1$.

2. Show that requiring that the functional

$$I[y] = \int_a^b \left(p(x)(y')^2 - q(x)y^2 \right) dx$$

to have an optimum value subject to the constraint

$$\int_a^b y^2 w(x) dx = 1$$

leads to the differential equation

$$\frac{d}{dx}(py') + qy - \lambda wy = 0$$

3. Solve the following problems from *Perfect Form* by Lemons: 4.5 and 4.6.