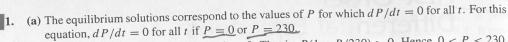
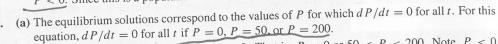
Blanchard and Devaney Ch 1.1, Ex 1,2,8,9,13,15,19

EXERCISES FOR SECTION 1.1



- (b) The population is increasing if dP/dt > 0. That is, P(1 P/230) > 0. Hence, 0 < P < 230.
- (c) The population is decreasing if dP/dt < 0. That is, P(1 P/230) < 0. Hence, P > 230 or P < 0. Since this is a population model, P < 0 might be considered "nonphysical."



(b) The population is increasing if dP/dt > 0. That is, P < 0 or 50 < P < 200. Note, P < 0 might be considered "nonphysical" for a population model.

(c) The population is decreasing if dP/dt < 0. That is, 0 < P < 50 or P > 200.

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They are both learning at the same rate when t = 0.

8. (a) The rate of change of the amount of radioactive material is dr/dt. This rate is proportional to the amount r of material present at time t. With $-\lambda$ as the (positive) proportionality constant, we obtain the differential equation $\frac{dr}{dt} = -\lambda r$

Note that the minus sign (along with the assumption that λ is positive) means that the material decays.

(b) The only additional assumption is the initial condition $r(0) = r_0$. Consequently, the corresponding initial-value problem is

 $\left(\frac{dr}{dt} = -\lambda r, \quad r(0) = r_0. \right)$

- 9. The general solution of the differential equation $dr/dt = -\lambda r$ is $r(t) = r_0 e^{-\lambda t}$ where $r(0) = r_0$ is the initial amount.
 - (a) We have $r(t) = r_0 e^{-\lambda t}$ and $r(5230) = r_0/2$. Thus

$$\frac{r_0}{2} = r_0 e^{-\lambda \cdot 5230}$$

$$\frac{1}{2} = e^{-\lambda \cdot 5230}$$

$$\ln\frac{1}{2} = -\lambda \cdot 5230$$

$$-\ln 2 = -\lambda \cdot 5230$$

because
$$\ln 1/2 = -\ln 2$$
. Thus,

$$\lambda = \frac{\ln 2}{5230} \approx 0.000132533.$$

- (b) We have $r(t) = r_0 e^{-\lambda t}$ and $r(8) = r_0/2$. By a computation similar to the one in part (a), we have $\lambda = \frac{\ln 2}{8} \approx 0.0866434.$
- (c) If r(t) is the number of atoms of C-14, then the units for dr/dt is number of atoms per year. Since $dr/dt = -\lambda r$, λ is "per year." Similarly, for I-131, λ is "per day." The unit of measurement of r does not matter.
- (d) We get the same answer because the original quantity, r_0 , cancels from each side of the equation. We are only concerned with the proportion remaining (one-half of the original amount).
- 13. Let P(t) be the population at time t, k be the growth-rate parameter, and N be the carrying capacity. The modified models are
 - (a) dP/dt = k(1 P/N)P 100
 - **(b)** dP/dt = k(1 P/N)P P/3
 - (c) $dP/dt = k(1 P/N)P a\sqrt{P}$, where a is a positive parameter.
- 14. (a) The differential equation is dP/dt = 0.3P(1 P/2500) 100. The equilibrium solutions of this equation correspond to the values of P for which dP/dt = 0 for all t. Using the quadratic formula, we obtain two such values, $P_1 \approx 396$ and $P_2 \approx 2104$. If $P > P_2$, dP/dt < 0, so P(t) is decreasing. If $P_1 < P < P_2$, dP/dt > 0, so P(t) is increasing. Hence the solution that satisfies the initial condition P(0) = 2500 decreases toward the equilibrium $P_2 \approx 2104$.
 - (b) The differential equation is dP/dt = 0.3P(1 P/2500) P/3. The equilibrium solutions of this equation are $P_1 \approx -277$ and $P_2 = 0$. If P > 0, dP/dt < 0, so P(t) is decreasing. Hence, for P(0) = 2500, the population decreases toward P = 0 (extinction).
- 15. Several different models are possible. Let R(t) denote the rhinoceros population at time t. The basic assumption is that there is a minimum threshold that the population must exceed if it is to survive. In terms of the differential equation, this assumption means that dR/dt must be negative if R is close to zero. Three models that satisfy this assumption are:
 - If k is a growth-rate parameter and M is a parameter measuring when the population is "too small", then

$$\int \frac{dR}{dt} = kR\left(\frac{R}{M} - 1\right).$$

• If k is a growth-rate parameter and b is a parameter that determines the level the population will start to decrease (R < b/k), then

$$\frac{dR}{dt} = kR - b.$$

• If k is a growth-rate parameter and b is a parameter that determines the extinction threshold, then

$$\frac{dR}{dt} = aR - \frac{b}{R}.$$

In each case, if R is below a certain threshold, dR/dt is negative. Thus, the rhinos will eventually die out. The choice of which model to use depends on other assumptions. There are other equations that are also consistent with the basic assumption.

- 19. (a) We consider dx/dt in each system. Setting y = 0 yields dx/dt = 5x in system (i) and dx/dt = x in system (ii). If the number x of prey is equal for both systems, dx/dt is larger in system (i). Therefore, the prey in system (i) reproduce faster if there are no predators.
 - (b) We must see what affect the predators (represented by the y-terms) have on dx/dt in each system. Since the magnitude of the coefficient of the xy-term is larger in system (ii) than in system (i), y has a greater effect on dx/dt in system (ii). Hence the predators have a greater effect on the rate of change of the prey in system (ii).
 - (c) We must see what affect the prey (represented by the x-terms) have on dy/dt in each system. Since x and y are both nonnegative, it follows that

$$-2y + \frac{1}{2}xy < -2y + 6xy,$$

and therefore, if the number of predators is equal for both systems, dy/dt is smaller in system (i). Hence more prey are required in system (ii) than in system (ii) to achieve a certain growth rate.