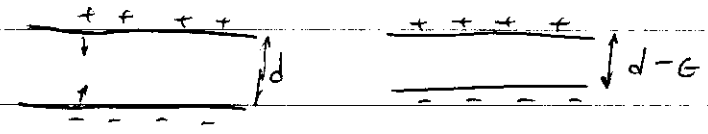
40/56 Suppose the plates of a parallel plate capacitor move closer together by a small distance ϵ as a result of their mutual attraction.

Plates have area A.



(a) Eqn $\frac{(2.52)}{103}$ Pressure on surface $P = \frac{\epsilon_0}{2} E^2 = \frac{\text{Energy}}{\text{volume}} = \frac{\text{force}}{\text{area}}$

(b) Energy lost by field = energy density × volume (2.46)

2.45 A sphere of radius R carries a charge density
107 $\rho(r) = kr$ where k = constant.
Find the energy of the configuration.

First find the charge enclosed by any radius.

$\rho = \frac{dq}{d\text{volume}} \rightarrow \int dq = \int \rho\, d\tau = \int_0^r kr\, 4\pi r^2\, dr$

INSIDE $q(r < R) =$

Total charge enclosed $Q =$

Now use Gauss's law to find the field everywhere
(this is easier than finding $V(q)$ or $V(\rho)$, thanks to
the symmetry)

$\oint E \cdot dA = \frac{q(r)}{\epsilon_0} = E \cdot 4\pi r^2 \rightarrow$ INSIDE $E(r<R) =$

$E(R) =$

outside $E(r>R)$

sketch $q(r)$ & $E(r)$

(2.45) continued... One way to find energy: $W = \frac{1}{2}\int \rho V d\tau$

Now, starting from the outside, find $V(r)$ everywhere

OUTSIDE: $V(r>R) = -\int_{\infty}^{r} E_{out}\, dr =$

At the surface: $V(R) =$

INSIDE: $V(r<R) = -\int_{\infty}^{R} E(r \geq R)\, dr - \int_{R}^{r} E_{in}\, dr =$

Finally, $W = \frac{1}{2}\int_{0}^{R} \rho V d\tau$ where $\rho = kr$ and $d\tau = 4\pi r^2 dr$

This is the energy stored in the charge distribution

Alternative: find the energy stored in the field:

$$W = \frac{\epsilon_0}{2}\int E^2 d\tau = \frac{\epsilon_0}{2}\int_0^R E_{in}^2 d\tau + \frac{\epsilon_0}{2}\int_R^\infty E_{out}^2 d\tau$$