EM HW #6 - Physical Systems - due 6 Mar 2007 - E/z

Ch 5 # $\underset{239}{26}$, $\underset{246}{33}$, $\underset{244}{38}$, $\underset{247}{39}$

$\underset{239}{26.}$ Find the $\vec{A}$ above and below the plane surface current in Ex 5.8

$\underset{226}{Ex\ 5.8}$ found by ampere's law that

$$\bar{B} = \frac{\mu_0 k}{2} \hat{y} \left(\begin{array}{l} + \text{ for } z < 0 \\ - \text{ for } z > 0 \end{array} \right.$$
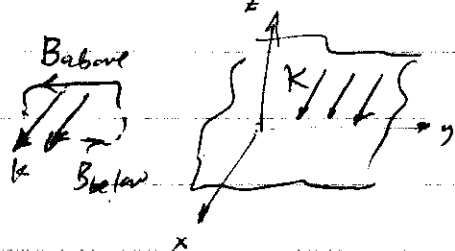
$$\vec{B} = \vec{\nabla} \times \vec{A} =$$

Problem 5.33 Show that the magnetic field of a dipole can be written in coordinate-free form:

$$\mathbf{B}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\mathbf{m}\cdot\hat{r})\hat{r} - \mathbf{m}]. \qquad (5.87)$$
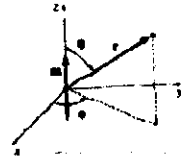

(r, θ, ϕ) is
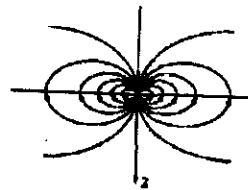
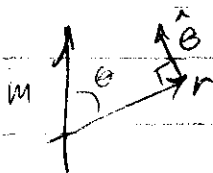$$\mathbf{A}_{dip}(\mathbf{r}) = \frac{\mu_0}{4\pi}\frac{m\sin\theta}{r^2}\hat{\phi}. \qquad (5.85)$$

and hence

$$\mathbf{B}_{dip}(\mathbf{r}) = \nabla \times \mathbf{A} = \frac{\mu_0 m}{4\pi r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}). \qquad (5.86)$$

The magnetic dipole moment

$\vec{m} = I\int d\vec{a} =$ can be written $\vec{m} = (\vec{m}\cdot\hat{r})\hat{r} + (\vec{m}\cdot\hat{e})\hat{e}$

where $\vec{m}\cdot\hat{r} = m\cos\Theta\,\hat{r}$ and $\vec{m}\cdot\hat{e} = \underline{\hspace{3cm}}$
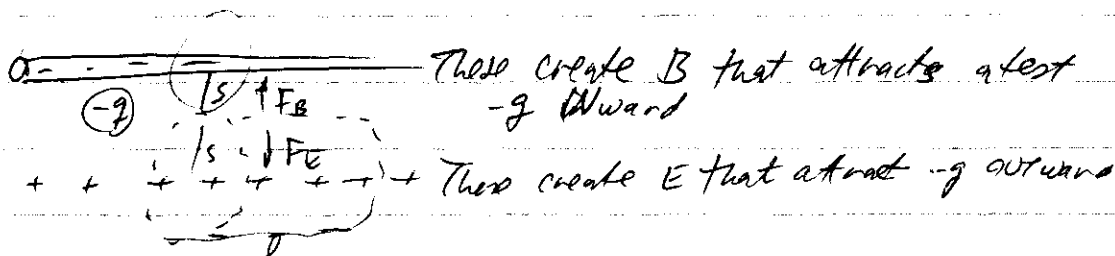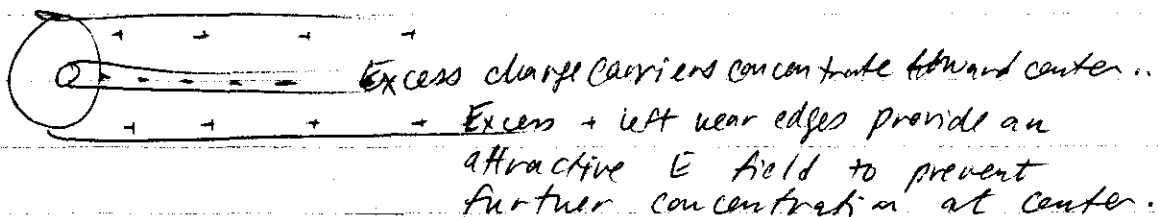
So $\vec{m} = m\cos\Theta\,\hat{r} + \underline{\hspace{3cm}}\hat{e}$

$3(\vec{m}\cdot\hat{r}) =$

So $3(\vec{m}\cdot\hat{r}) - \vec{m} =$

Given in spherical coordinates (5.86):

we can rewrite this in coordinate-free form as (5.87):

It may have occurred to you that since parallel currents attract, the current within a single wire should contract into a tiny concentrated stream along the axis. Yet in practice the current typically distributes itself quite uniformly over the wire. How do you account for this? If the positive charges (density ρ_+) are at rest, and the negative charges (density ρ_-) move at speed v (and none of these depends on the distance from the axis), show that $\rho_- = -\rho_+\gamma^2$, where $\gamma \equiv 1/\sqrt{1-(v/c)^2}$ and $c^2 = 1/\mu_0\epsilon_0$. If the wire as a whole is neutral, where is the compensating charge located?[16] [Notice that for typical velocities (see Prob. 5.19) the two charge densities are essentially unchanged by the current (since $\gamma \approx 1$). In plasmas, however, where the positive charges are *also* free to move, this so-called pinch effect can be very significant.]

Excess charge carriers concentrate toward center...

Excess + left near edges provide an attractive E field to prevent further concentration at center.

These create B that attracts a test $-q$ inward

These create E that attract $-q$ outward

Equilibrium : $F_B = F_E$.

Find $B(s)$: $\oint B \cdot dl = \mu_0 I = \mu_0 J \cdot$ cross sectional area of Ampérian loop , $J = \underline{\quad}\rho_-$

$B(s) =$

Find $E(s)$ $\oint E \cdot da = E \cdot 2\pi s \cdot l = \dfrac{q}{\epsilon_0}$ where $\rho = \dfrac{q}{\text{volume of Gaussian Can}}$, $\rho = \rho_+ + \rho_-$
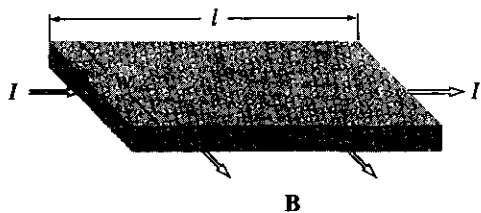
$E(s) =$

Problem 5.39 A current I flows to the right through a rectangular bar of conducting material, in the presence of a uniform magnetic field **B** pointing out of the page (Fig. 5.56).

(a) If the moving charges are *positive*, in which direction are they deflected by the magnetic field? This deflection results in an accumulation of charge on the upper and lower surfaces of the bar, which in turn produces an electric force to counteract the magnetic one. Equilibrium occurs when the two exactly cancel. (This phenomenon is known as the **Hall effect**.)

(b) Find the resulting potential difference (the **Hall voltage**) between the top and bottom of the bar, in terms of B, v (the speed of the charges), and the relevant dimensions of the bar.[17]

(c) How would your analysis change if the moving charges were *negative*? [The Hall effect is the classic way of determining the sign of the mobile charge carriers in a material.]

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} = 0 \text{ when}$$

$$\vec{E} = -\vec{v} \times \vec{B} = -\nabla V$$

$$\underset{\text{velocity}}{\uparrow} \qquad \underset{\text{voltage}}{\uparrow}$$

$$\Delta V = -\int \vec{E} \cdot d\ell = \int (\vec{v} \times \vec{B}) \cdot d\ell = vBt$$