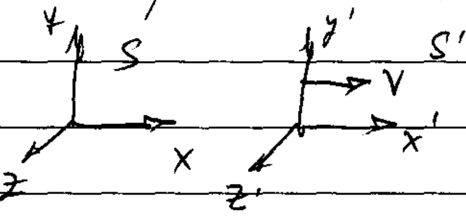
Modern Physics Ch 1 HW due 25 Jan 2007

E1 Zta

Derive γ, #8, 44, 49-50

Derive γ for (relativistic) Lorentz transformation between x-coordinate in rest frame S and x' coordinate in frame S' moving with speed v in x direction



Galilean: $x' = x - vt$

(1-13) Lorentz: $x' = \gamma(x - vt)$

(1-14) Inverse: $x = \gamma(x' + vt')$

First, solve (1-13) and (1-14) for t' by eliminating x':

$$x = \gamma[\gamma(x-vt) + vt'] = \gamma^2(x-vt) + \gamma vt'$$

$$\gamma vt' = -\gamma^2(x-vt) + x$$

$$t' = -\gamma\frac{(x-vt)}{v} + \frac{x}{\gamma v} = \gamma\left[t + \frac{x}{v}\left(\frac{1}{\gamma^2} - 1\right)\right] =$$

(1-15) $$t' = \gamma\left[t + \frac{x}{v}\left(\frac{1-\gamma^2}{\gamma^2}\right)\right] \checkmark$$

This transformation must work for a flash of light at the origin in either the

(1-16) rest frame $x^2+y^2+z^2 = c^2t^2$ or the

(1-17) moving frame $x'^2+y'^2+z'^2 = c^2t'^2$ (note $y'=y$, $z'=z$)

Substitute (1-13) and (1-15) into (1-17):

$$\gamma^2(x-vt)^2 + \overset{=y^2}{y'^2} + \overset{=z^2}{z'^2} = c^2\gamma^2\left[t + \frac{x}{v}\left(\frac{1-\gamma^2}{\gamma^2}\right)\right]^2$$

(1) $\gamma^2(x^2-2xvt+v^2t^2)+y^2+z^2 = c^2\gamma^2\left[t^2+\frac{x^2}{v^2}\left(\frac{1-\gamma^2}{\gamma^2}\right)^2 + 2t\frac{x}{v}\left(\frac{1-\gamma^2}{\gamma^2}\right)\right]$

(2) This must transform to (1-16) $x^2+y^2+z^2=c^2t^2$

Let's just match the x^2 terms in (1) and (2)

(1) (2)

$x^2 = x^2\gamma^2 - c^2\gamma^2\frac{x^2}{v^2}\left(\frac{1-\gamma^2}{\gamma^2}\right)^2$

(1-8.) Consider two [partially covered] frames. When an observer in each frame measures the following quanti[...] same results? Explai[...]

(a) The distance be[...]
(b) The value of the[...]
(c) The speed of lig[...]
(d) The time interv[...]
(e) Newton's first l[...]
(f) The order of the[...]
(g) The value of the[...]

1-44. H. A. Lorentz suggested 15 years before Einstein's 1905 paper that the null effect of the Michelson-Morley experiment could be accounted for by a contraction of that arm of the interferometer lying parallel to Earth's motion through the ether to a length $L = L_p(1 - v^2/c^2)^{-1/2}$. He thought of this, incorrectly, as an actual shrinking of matter. By about how many atomic diameters would the material in the parallel arm of the interferometer have had to shrink in order to account for the absence of the expected shift of 0.4 of a fringe width? (Assume the diameter of atoms to be about 10^{-10} m.)

L_p = proper Length

$L = \frac{L_p}{\gamma}$ = contracted

$$\Delta L = L_p - L = L_p - \frac{L_p}{\gamma} = L_p\left(1 - \frac{1}{\gamma}\right)$$

Binomial expansion: $\frac{1}{\gamma} = \sqrt{1 - \frac{v^2}{c^2}} = \left(1 - \frac{v^2}{c^2}\right)^{\frac{1}{2}}$

$$= 1 + \frac{1}{2}\left(-\frac{v^2}{c^2}\right) + \frac{1}{2}\left(-\frac{1}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \ldots$$

$$= 1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\frac{v^4}{c^4} + \ldots$$

Since $v_{earth} << c$, approximate $\frac{1}{\gamma} \approx 1 - \frac{1}{2}\frac{v^2}{c^2}$

then $\left(1 - \frac{1}{\gamma}\right) \approx 1 - \left(1 - \frac{1}{2}\frac{v^2}{c^2}\right) =$ _____

p.12 Length of interferometer arms L_p = _____ m

p.9 Speed of Earth "through ether" v = _____ m/s

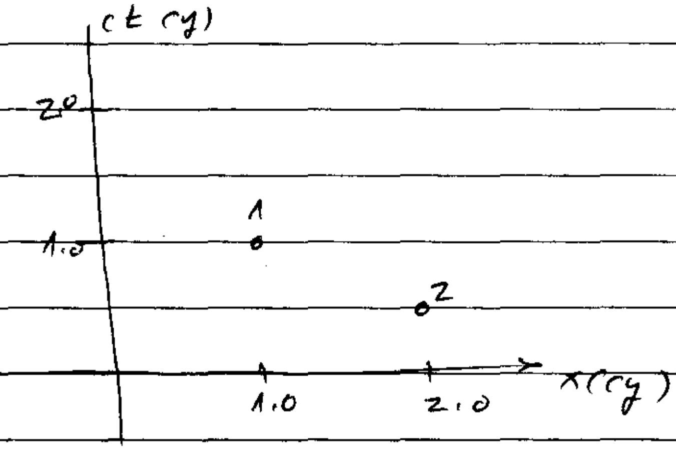
$\left(1 - \frac{1}{\gamma}\right) =$ _____

$\Delta L =$ _____

"Shrinkage" = $\frac{\Delta L}{10^{-10}\text{ m/atomic diameter}}$ = _____ atomic diam

1-49. Frames S and S' are moving relative to each other along the x and x' axes. They set their clocks to $t = t' = 0$ when their origins coincide. In frame S, event 1 occurs at $x_1 = 1\ c \cdot y$ and $t_1 = 1$ y and event 2 occurs at $x_2 = 2.0\ c \cdot y$ and $t_2 = 0.5$ y. These events occur simultaneously in frame S'. (*a*) Find the magnitude and direction of the velocity of S' relative to S. (*b*) At what time do both of these events occur as measured in S'? (*c*) Compute the spacetime interval Δs between the events. (*d*) Is the interval spacelike, timelike, or lightlike? (*e*) What is the proper distance L_p between the events?

1-50. Do Problem 1-49 parts (*a*) and (*b*) using a spacetime diagram.



(a)

(1-22) Time dilation $\Delta t' = \gamma \Delta t - \frac{\gamma v}{c^2} \Delta x$

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Simultaneous in S': $\Delta t' = 0 \rightarrow \Delta t' = 0 \rightarrow \gamma \Delta t = \frac{\gamma v}{c^2} \Delta x$

Solve for v:

(b) At what time do these events occur as measured in S'?

(1-20) $t' = \gamma \left(t - \frac{vx}{c^2}\right)$ $\gamma =$ ______

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$t' =$

(c) Spacetime interval ΔS

(d)

(e) $L_p = \Delta S$

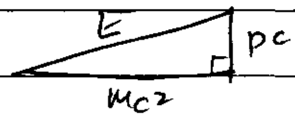
Modern Physics Ch 2 - Relativity II #13, 35, 48

13. The total energy of a particle is twice its rest energy. (*a*) Find *u*/*c* for the particle. (*b*) Show that its momentum is given by $p = (3)^{1/2}\,mc$.

$$E = \gamma mc^2 \rightarrow \gamma = \frac{E}{mc^2} = \_\_\_\_$$

$$\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \rightarrow \text{find } \frac{u}{c}$$

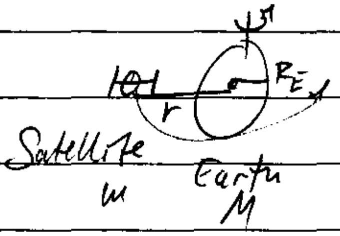
(b)



$p =$

2-35.) A synchronous satellite "parked" in orbit over the equator is used to relay microwave transmissions between stations on the ground. To what frequency must the satellite's receiver be tuned if the frequency of the transmission from Earth is exactly $f_0 =$ 9.375 GHz? (Ignore all Doppler effects.)

Find the radius and g for geosynchronous orbits : $T = 1$ day



① $F = \frac{GmM}{r^2} = \frac{mv^2}{r} \rightarrow v^2 =$ ______

② $v = \frac{2\pi r}{T}$

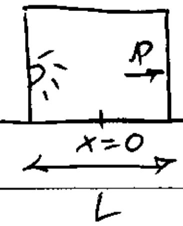
Eliminate v from ① & ②, solve for r = satellite orbit radius

Then you can find g(r) at the satellite orbit radius from $\frac{GmM}{r^2} = mg$
where M = mass of Earth, G = grav. constant

Once you have g(r), you can use the general relativistic frequency shift $\Delta f = f_0 - f = \frac{f_0 gh}{c^2}$ (2-45) 111

Where h = altitude = $r - R_{Earth}$

2-48. In a simple thought experiment, Einstein showed that there is mass associated with electromagnetic radiation. Consider a box of length L and mass M resting on a frictionless surface. At the left wall of the box is a light source that emits radiation of energy E, which is absorbed at the right wall of the box. According to classical electromagnetic theory, this radiation carries momentum of magnitude $p = E/c$. (*a*) Find the recoil velocity of the box such that momentum is conserved when the light is emitted. (Since p is small and M is large, you may use classical mechanics.) (*b*) When the light is absorbed at the right wall of the box, the box stops, so the total momentum remains zero. If we neglect the very small velocity of the box, the time it takes for the radiation to travel across the box is $\Delta t = L/c$. Find the distance moved by the box in this time. (*c*) Show that if the center of mass of the system is to remain at the same place, the radiation must carry mass $m = E/c^2$.



(a) $P_{light} = \frac{E}{c}$ $\quad P_{box} = MV \rightarrow V =$

(b) $\Delta x = V\Delta t$, $\Delta t = \frac{L}{c}$

$\Delta x =$

(c) Say radiation of mass m is emitted from the left side of the box at $x = -\frac{L}{2}$. Then the center of mass of the box is at

$$x_{cm_1} = \frac{\sum m_i x_i}{\sum m_i} = \frac{M\cdot 0 + m(\frac{L}{2})}{M+m} =$$

When the radiation is absorbed on the right side of the box, the box has moved a bit, and the center of mass is now at

$$x_{cm_2} =$$

Since this is an internal process, the center of mass cannot move: $x_{cm_1} = x_{cm_2}$. Solve for m: