Modern Physics HW due week 4 Thurs 1 Feb

Ch 3 # 15, 23, 58, 25 E/Z/Zn

Ch 4 # 3, 17, 24

3-15. As noted in the chapter, the cosmic microwave background radiation fits the Planck equations for a blackbody at 2.7 K. (*a*) What is the wavelength at the maximum intensity of the spectrum of the background radiation? (*b*) What is the freqnency of the radiation at the maximum? (*c*) What is the total power incident on Earth from the background radiation?

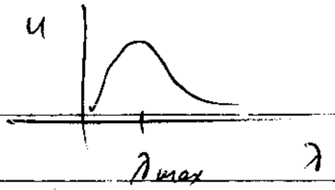
$\lambda_p T = 3 \times 10^{-3}$ mK

(b) $E = \frac{hc}{\lambda} = hf$

(c) $\frac{Power}{area} = \sigma T^4$ is incident on an effective area of a flat DISK of radius $= R_{earth}$, due to projection effects (more at equator, less at poles)

3.23. Use Planck's Law $u(\lambda) = \frac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$ $\underline{(3.33)}$ p.138 to derive the Constant in Wien's Law

$\lambda_{peak} T = 3\cdot 10^{-3}$ m·K $\underline{(3.20)}$ p.134



Let $u = \frac{A\lambda^{-5}}{e^{B/\lambda} - 1}$. We want to find where u peaks, or where $\frac{\partial u}{\partial \lambda} = 0$. Solve for λ.

$\frac{\partial u}{\partial \lambda} =$

3-25. The orbiting space shuttle moves around Earth well above 99 percent of the atmosphere, yet it still accumulates an electric charge on its skin due, in part, to the loss of electrons caused by the photoelectric effect with sunlight. Suppose the skin of the shuttle is coated with Ni, which has a relatively large work function $\phi = 4.87$ eV at the temperatures encountered in orbit. (*a*) What is the maximum wavelength in the solar spectrum that can result in the emission of photoelectrons from the shuttle's skin? (*b*) What is the maximum fraction of the total power falling on the shuttle that could potentially produce photoelectrons?

Photoelectric effect   Energy$_{in}$ = Energy$_{out}$

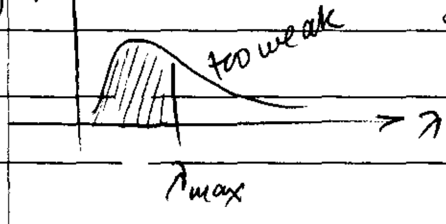
$$\text{Light} = hf = \underbrace{\tfrac{1}{2}mv^2}_{\text{kinetic}} + \underbrace{\phi}_{\text{work functn}}$$

If you turn on a repelling potential, V_0, it limits the maximum kinetic energy of emitted particles:

$$eV_0 = \left(\tfrac{1}{2}mv^2\right)_{max} = hf - \phi$$

(a) With no stopping potential, consider barely ejected electrons with $KE \to 0$. Then $hf = \frac{hc}{\lambda} \to \phi$

$\lambda_{max} =$

(b) 

Sun's $\frac{\text{Power}}{\text{area}}$ curve

If λ_{max} is the largest wavelength that can photo-eject electrons,

then $f = \frac{\text{Power in sufficiently energetic photons}}{\text{total incident power}} = \frac{\text{Shaded area}}{\text{total area under curve}}$

total area $= R = \sigma T^4 =$

Shaded area $\approx u(\lambda)\Delta\lambda$ where $\Delta\lambda \sim \lambda_{max}$ and $u\left(\frac{\lambda_{max}}{2}\right)$

3-58. Derive (3.32) from (3.30) and (3.31) p.138

$$\sum_{n=0}^{\infty} f_n = A \sum_{n=0}^{\infty} e^{-n\epsilon/kT} = 1 \qquad (3\text{-}30)$$

The average energy of an oscillator is then given by the discrete-sum equiv
Equation 3-27,

$$\overline{E} = \sum_{n=0}^{\infty} E_n f_n = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} \qquad (3\text{-}31)$$

Calculating the sums in Equations 3-30 and 3-31 (see Problem 3-58) yields th

$$\overline{E} = \frac{\epsilon}{e^{\epsilon/kT} - 1} = \frac{hf}{e^{hf/kT} - 1} = \frac{hc/\lambda}{e^{hc/\lambda kT} - 1} \qquad (3\text{-}32)$$

Multiplying this result by the number of oscillators per unit volume in the inte
given by Equation 3-23, we obtain for the energy density distribution functior
radiation in the cavity: $(3\text{-}33)\ u(\lambda) = \dfrac{8\pi h c\, \lambda^{-5}}{e^{hc/\lambda kT} - 1}$

$(3\text{-}23)$ p.136 $u(\lambda) = \dfrac{8\pi}{\lambda^4}$

$(3\text{-}29)\quad f_n = A e^{-E_n/kT} = A e^{-n\epsilon/kT}$

$E_n = n\epsilon = nhf$ where n = integer

ϵ = quantized energy

f = quantized frequency

h = Planck constant

Let $e^{-n\epsilon/kT} = e^{-nhf/kT} = e^{-nx}$ $\quad (x = hf/kT)$

Then $\sum_{n=0}^{\infty} f_n = A \sum_{n=0}^{\infty} e^{-nx} = A[e^0 + e^{-x} + (e^{-x})^2 + \ldots]$

$= A[1 + y + y^2 + \ldots] \equiv 1$ by definition

$(y = e^{-x})$

Trick: Compare this to the power series expansion for $(1-y)^{-1}$

$(1+z)^p = 1 + pz + \frac{p(p-1)}{2!} z^2 + \ldots$

$(1-y)^{-1} =$

Then $\sum f_n = A(1-y)^{-1} \equiv 1 \rightarrow A =$ ______

$$(3\text{-}31)\quad \bar{E} = \sum_{n=0}^{\infty} E_n A e^{-E_n/kT} = A\sum_{n=0}^{\infty} (\underset{E_n}{\uparrow}\qquad) e^{-nhf/kT} = Ahf\sum_{n=0}^{\infty} n e^{-nx}$$

Trick: $\frac{d}{dx} e^{-nx} =$ ______ and we found $\sum e^{-nx} = (1-y)^{-1}$

Combine these to get $\sum n e^{-nx} = -\frac{d}{dx}\sum e^{-nx} = -\frac{d}{dx}(1-y^{-1})$

Evaluate in general and simplify

$-\frac{d}{dx}(1-y^{-1}) =$

Then use $\frac{dy}{dx} = \frac{d}{dx}(e^{-x}) =$ ______

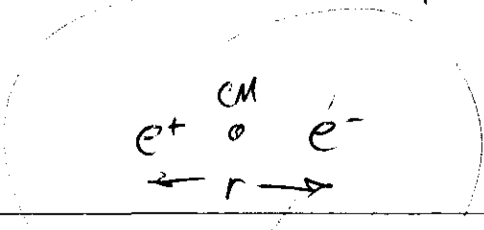
Sub into (3-31) and show that $\bar{E} = \frac{hfy}{1-y}$:

Sub in $y = e^{-x}$, multiply by $\frac{e^{-x}}{e^{-x}}$, Substitute in $x = {}^{hf}/_{kT}$ to get result :

POSITRONIUM

MUTUAL ORBIT p. 98

4-24. The electron-positron pair that was discussed in Chapter 2 can form a hydrogenlike system called *positronium*. Calculate (*a*) the energies of the three lowest states and (*b*) the wavelength of the Lyman α and β lines. (Detection of those lines is a "signature" of positronium formation.)



The easy way: just use $E_n = -\frac{hcR}{n^2}$ (4-23)

and the reduced mass correction

to the Rydberg constant: $R = R_\infty \left(\frac{1}{1+\frac{m}{M}} \right) =$ ______

Or, a real derivation of the energy levels, using the reduced mass $\mu = \frac{m \cdot m}{m+m} =$ ______

(As done in class!)

$\frac{1}{2}\mu v^2 = \frac{kqQ}{r} = \frac{ke^2}{r}$ $\qquad$ $L = n\hbar = \mu v r$

$v^2 =$ $\qquad\qquad$ $v^2 =$

Eliminate v^2 and solve for r

Substitute r into (virial theorem!) $E_{tot} = \frac{1}{2}V = -\frac{1}{2}\frac{ke^2}{r}$

$E_{tot} = E_n =$

Either way, you should get half the energy of H atom.

(a) $E_1 =$

$E_2 = \dfrac{E_1}{2^2}$

$E_3 = \dfrac{E_1}{3^2}$

(b) Lyman series lines are from transitions to the $n=1$ level.

Lyman α: $E_2 \rightarrow E_1$ Lyman β: $E_3 \rightarrow E_1$