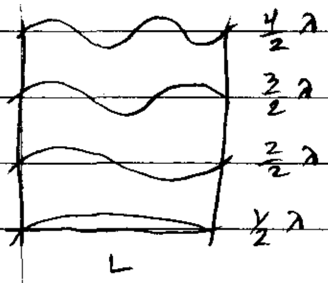
Modern Physics HW due 13 Feb 2007 - [illegible]

Ch 5 # 7, 19, 24, 27, 30, 34

5-7. A free proton moves back and forth between rigid walls separated by a distance $L = 0.01$ nm. (*a*) If the proton is represented by a one-dimensional standing de Broglie wave with a node at each wall, show that the allowed values of the de Broglie wavelength are given by $\lambda = 2L/n$ where n is a positive integer. (*b*) Derive a general expression for the allowed kinetic energy of the proton and compute the values for $n = 1$ and 2.

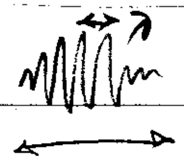


(a)

(b) $E = \frac{hc}{\lambda} = pc$

$KE = \frac{p^2}{2m}$

5-19. A radar transmitter used to measure the speed of pitched baseballs emits pulses of $\lambda =$ 2.0-cm wavelength that are 0.25 μs in duration (Δt). (*a*) What is the length of the wave packet produced? $= \Delta X$ (*b*) To what frequency should the receiver be tuned? (*c*) What must be the minimum bandwidth of the receiver?



$\Delta X, \Delta t$

$c = f\lambda$ so the radar frequency is

$f = \qquad\qquad = \frac{1}{\text{period}} = \frac{1}{T}$

The number of waves in each packet is

$N = \frac{\Delta t}{T} =$

The length of the wave packet $\Delta X = N\lambda$

(b) Is the receiver sensitive to the signal frequency f or the pulse frequency $\frac{1}{\Delta t}$?

(c) Bandwidth: What is the minimum frequency spread $\Delta\omega$ that will permit localization of the wave packet in time to Δt?

$\Delta E \Delta t \geq \hbar$ and $E = \hbar f$ where $\omega = 2\pi f$

5-24. **A particle moving in one dimension between rigid walls separated by a distance L has the wave function $\Psi(x) = A \sin(\pi x/L)$. Since the particle must remain between the walls, what must be the value of A?**

Probability of being between the walls $= 1 = \int_0^L \Psi^2 \, dx$

$1 = A^2 \int_0^L \sin^2 \frac{\pi x}{L} \, dx$ Either look it up or use trig identity

[illegible] 27 If an excited state of an atom is known to have a lifetime of 10^{-7} s, what is the uncertainty in the energy of photons emitted by such atoms in the spontaneous decay to the ground state?

$\Delta E \Delta t \geq \hbar$

5-30. If the uncertainty in the position of a wave packet representing the state of a quantum-system particle is equal to its de Broglie wavelength, how does the uncertainty in momentum compare with the value of the momentum of the particle?

deBroglie wavelength $E = \frac{hc}{\lambda} = pc \rightarrow p = \frac{h}{\lambda} = mv \rightarrow \lambda = \frac{h}{p}$

$\Delta x \Delta p \geq \hbar$

5-34. A neutron has a kinetic energy of 10 MeV. What size object is necessary to observe neutron diffraction effects? Is there anything in nature of this size that could serve as a target to demonstrate the wave nature of 10-MeV neutrons?

Need an object ~ the size of the neutron's deBroglie wavelength.

$\rightarrow$ $E_{tot} = \gamma mc^2 = \sqrt{(mc^2)^2 + (pc)^2}$ and $\frac{hc}{\lambda} = pc \rightarrow p = \gamma mv = \frac{h}{c}$

Solve for γ and v, using $mc^2_{neutron} = 939$ MeV

Then find λ: