Quantum mechanics HW 1a, Tuesday 3 April 2007

1.1.4, 1.6.1, 1.6.2, 1.6.3

*Exercise 1.1.4.** Consider the vector space discussed in Exercise 1.1.1. Show that the elements (1, 1, 0), (1, 0, 1), and (3, 2, 1) are linearly dependent. [Assume that one of them is a linear combination of the other two, and find the (nontrivial) coefficients of the expansion.] Show likewise that (1, 1, 0), (1, 0, 1), and (0, 1, 1) are LI.

Theorem 5 (Gram–Schmidt Theorem). Given n vectors $|V_1\rangle, |V_2\rangle, \ldots, |V_n\rangle$ that are LI, we can get, by forming linear combinations, n orthonormal vectors, $|1\rangle, |2\rangle, \ldots, |i\rangle, \ldots, |n\rangle$.

Proof. Let us first construct n mutually *orthogonal* vectors. Let

$$|1'\rangle = |V_1\rangle$$

$$|2'\rangle = |V_2\rangle - \frac{|1'\rangle\langle 1'|V_2\rangle}{\langle 1'|1'\rangle}$$

Exercise 1.3.2. Consider the vectors

$$|V_1\rangle \leftrightarrow \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \qquad |V_2\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \qquad |V_3\rangle \leftrightarrow \begin{bmatrix} 0 \\ 2 \\ 5 \end{bmatrix}$$

Use the Gram–Schmidt procedure to get the following orthonormal basis:

$$|1\rangle \leftrightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad |2\rangle \leftrightarrow \begin{bmatrix} 0 \\ 1/5^{1/2} \\ 2/5^{1/2} \end{bmatrix}, \qquad |3\rangle \leftrightarrow \begin{bmatrix} 0 \\ -2/5^{1/2} \\ 1/5^{1/2} \end{bmatrix}$$

Is this the only orthonormal basis you can get in this case? (What if you change the sign of the components of $|1\rangle$?)

Exercise 1.6.1. An operator Ω is given by the matrix

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

What is its action? What [illegible] A [illegible]?

*Exercise 1.6.2.** Given Ω and Λ are Hermitian what can you say about (i) $\Omega\Lambda$; (ii) $\Omega\Lambda + \Lambda\Omega$; (iii) $[\Omega, \Lambda]$; and (iv) $i[\Omega, \Lambda]$?

*Exercise 1.6.3.** Show that a product of unitary operators is unitary.