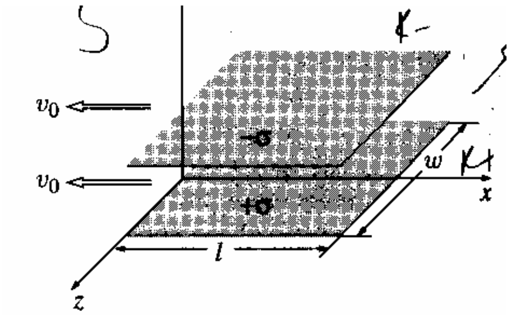
EM HW 8b Thus. 24 May 2007 Griffiths **Ch.12** # 41, 43, 44

Problem 12.41 Why can't the electric field in Fig. 12.35b have a z component? After all, the *magnetic* field does.



Problem 12.43

(a) Check that Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{a} = (1/\epsilon_0)Q_{\text{enc}}$, is obeyed by the field of a point charge in uniform motion, by integrating over a sphere of radius R centered on the charge.

(b) Find the Poynting vector for a point charge in uniform motion. (Say the charge is going in the z direction at speed v, and calculate $\mathbf{S}$ at the instant q passes the origin.)

Problem 12.44

(a) Charge q_A is at rest at the origin in system S; charge q_B flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

(b) Now study the same problem from system $\bar{S}$, which moves to the right with speed v. What is the force on q_B when q_A passes the $\bar{y}$ axis? [Do it two ways: (i) by using your answer to (a) and transforming the force; (ii) by computing the fields in $\bar{S}$ and using the Lorentz force law.]

Problem 12.45 Two charges $\pm q$, are on parallel trajectories a distance d apart, moving with equal speeds v in opposite directions. We're interested in the force on $+q$ due to $-q$ at the instant they cross (Fig. 12.42). Fill in the following table, doing all the consistency checks you can think of as you go along.

	System A (Fig. 12.42)	System B ($+q$ at rest)	System C ($-q$ at rest)
E at $+q$ due to $-q$:			
B at $+q$ duc to $-q$:			
F on $+q$ due to $-q$:			

Problem 12.43

(a) Check that Gauss's law, $\int \mathbf{E} \cdot d\mathbf{a} = (1/\epsilon_0)Q_{enc}$, is obeyed by the field of a point charge in uniform motion, by integrating over a sphere of radius R centered on the charge.

(12.92) $E = \frac{1}{4\pi\epsilon_0} \frac{q(1-\frac{v^2}{c^2})}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}} \frac{\hat{R}}{R^2}$ $\quad \int E \cdot d\vec{a} = \int E \cdot R^2 \sin\theta \, d\theta \, d\phi$

$= 2\pi \int_0^\pi E \sin\theta \, d\theta$

$\int E \cdot d\vec{a} = \frac{2\pi}{4\pi\epsilon_0} \frac{R^2}{R^2} q(1-\frac{v^2}{c^2}) \int_0^\pi \frac{\sin\theta \, d\theta}{(1-\frac{v^2}{c^2}\sin^2\theta)^{3/2}}$

let $u = \cos\theta$

$du = -\sin\theta \, d\theta$

$\sin^2\theta = 1-\cos^2\theta = 1-u^2$

$1-\frac{v^2}{c^2}\sin^2\theta = 1-\frac{v^2}{c^2}(1-u^2) = 1-\frac{v^2}{c^2}+\frac{v^2}{c^2}u^2$

$= \frac{v^2}{c^2}(\frac{c^2}{v^2}-1+u^2)$

$\int E \cdot da = \frac{q(1-\frac{v^2}{c^2})}{2\epsilon_0} \int_{-1}^{1} \frac{-du}{(\frac{v^2}{c^2})^{3/2}(\frac{c^2}{v^2}-1+u^2)^{3/2}} = \frac{-q(1-\frac{v^2}{c^2})}{2\epsilon_0}\left(\frac{c}{v}\right)^3 \int_{-1}^{1} \frac{du}{(a^2+u^2)^{3/2}}$

where $a^2 = \frac{c^2}{v^2}-1$. $\int_{-1}^{1} \frac{du}{(a^2+u^2)^{3/2}} =$

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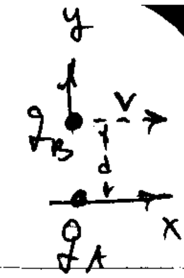
(b) Find the Poynting vector for a point charge in uniform motion. (Say the charge is going in the z direction at speed v, and calculate S at the instant q passes the origin.)

(12.11) Example 12.14 p.532 $\quad \mathbf{B} = \frac{\mu_0}{4\pi} \frac{qv\left(1-\frac{v^2}{c^2}\right)\sin\theta}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} R^2} \hat{\phi}$

(12.92) Example 12.13 p.527-8 $\quad \mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q\left(1-\frac{v_0^2}{c^2}\right)}{\left[1-\frac{v_0^2}{c^2}\sin^2\theta\right]^{3/2} R^2} \hat{R}$

$\mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0}$, $\hat{R}\times\hat{\phi} = -\hat{\theta}$

Problem 12.44



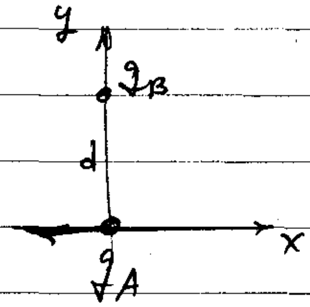
(a) Charge q_A is at rest at the origin in system S; charge q_B flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

(b) Now study the same problem from system $\bar{S}$, which moves to the right with speed v. What is the force on q_B when q_A passes the $\bar{y}$ axis? [Do it two ways: (i) by using your answer to (a) and transforming the force; (ii) by computing the fields in $\bar{S}$ and using the Lorentz force law.]

(a) Field of q_A at $y=d$ is $\mathbf{E}_A = \frac{1}{4\pi\epsilon_0}\frac{q_A}{d^2}\hat{y}$, $\mathbf{B}=0$

Force on q_B is $\mathbf{F}_B = q_B \mathbf{E}_A =$

(b)



Now that q_A is moving, its field at $y=d$ is given by (12.92) 527-8

$$\mathbf{E}_A = \frac{q_A}{4\pi\epsilon_0}\frac{\left(1-\frac{v^2}{c^2}\right)}{\left[1-\frac{v^2}{c^2}\sin^2\theta\right]^{3/2} d^2}\hat{y}$$

$\theta = 90°$

Simplify

$\mathbf{E}_A =$

What is $\mathbf{B}_A$? Is it zero? If not, what will be the magnetic force on q_B?

$F_B =$

Problem 12.46

(a) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant.

(b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant.

(c) Suppose that in one inertial system $\mathbf{B} = 0$ but $\mathbf{E} \neq 0$ (at some point P). Is it possible to find another system in which the *electric* field is zero at P?

For motion along x with speed v,

$\frac{(12.108)}{531}$ $\bar{E}_x = E_x \quad \bar{E}_y = \gamma(E_y - vB_z) \quad \bar{E}_z = \gamma(E_z + vB_y)$

$\bar{B}_x = B_x \quad \bar{B}_y = \gamma\left(B_y + \frac{v}{c^2}E_z\right) \quad \bar{B}_z = \gamma\left(B_z - \frac{v}{c^2}E_y\right)$

(a) $(\mathbb{E} \cdot \mathbb{B}) = E_x B_x + E_y B_y + E_z B_z$. (In the frame at rest)

In the moving frame,

$\mathbb{E} \cdot \mathbb{B} =$

Problem 12.47 An electromagnetic plane wave of (angular) frequency ω is traveling in the x direction through the vacuum. It is polarized in the y direction, and the amplitude of the electric field is E_0.

(a) Write down the electric and magnetic fields, $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$. [Be sure to define any auxiliary quantities you introduce, in terms of ω, E_0, and the constants of nature.]

(b) This same wave is observed from an inertial system $\bar{S}$ moving in the x direction with speed v relative to the original system S. Find the electric and magnetic fields in $\bar{S}$, and express them in terms of the $\bar{S}$ coordinates: $\bar{\mathbf{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ and $\bar{\mathbf{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$. [Again, be sure to define any auxiliary quantities you introduce.]

(c) What is the frequency $\bar{\omega}$ of the wave in $\bar{S}$? Interpret this result. What is the wavelength $\bar{\lambda}$ of the wave in $\bar{S}$? From $\bar{\omega}$ and $\bar{\lambda}$, determine the speed of the waves in $\bar{S}$. Is it what you expected?

For motion in the z direction

(9.48) p 370 $\mathbf{E}(z,t) = E_0 \cos(kz - \omega t)\,\hat{x}$ $\quad \mathbf{B}(z,t) = \frac{E_0}{c}\cos(kz - \omega t)\,\hat{y}$ $\quad k = \frac{\omega}{c}$

For a wave traveling in the x direction:

$\mathbf{E}(x,t) =$ $\quad \mathbf{B}(x,t) =$

(b) Transform the fields with (12.108) as usual; let $\alpha = \gamma(1 - \frac{v}{c}) = \sqrt{\frac{1-v/c}{1+v/c}}$

$\bar{E}_x =$ $\quad \bar{E}_y =$

$\bar{B}_x =$ $\quad \bar{B}_z =$

Inverse Lorentz transformation (12.19) $x = \gamma(\bar{x} + v\bar{t})$, $t = \gamma(\bar{t} + \frac{v}{c^2}\bar{x})$

$kx - \omega t =$

$\bar{\mathbf{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t}) =$

$\bar{\mathbf{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t}) =$