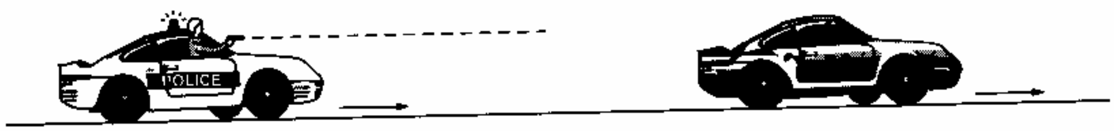
Physical Systems – spring 2007

EM HW 8a Tues. 22 May 2007 Griffiths **Ch.12** # 4, 13, 15, 20, 26, 34 (optional: #11)

EM HW 8b Thus. 24 May 2007 Griffiths **Ch.12 #** 41, 43, 44

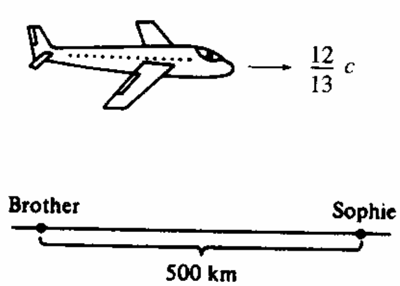
Problem 12.4 As the outlaws escape in their getaway car, which goes $\frac{3}{4}c$, the police officer fires a bullet from the pursuit car, which only goes $\frac{1}{2}c$ (Fig. 12.3). The muzzle velocity of the bullet (relative to the gun) is $\frac{1}{3}c$. Does the bullet reach its target (a) according to Galileo, (b) according to Einstein?



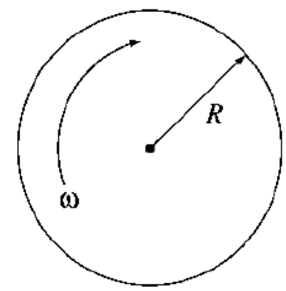
Problem 12.15 You probably did Prob. 12.4 from the point of view of an observer on the *ground*. Now do it from the point of view of the police car, the outlaws, and the bullet. That is, fill in the gaps in the following table:

speed of → / relative to ↓	Ground	Police	Outlaws	Bullet	Do they escape?
Ground	0	$\frac{1}{2}c$	$\frac{3}{4}c$		
Police				$\frac{1}{3}c$	
Outlaws					
Bullet					

Problem 12.13 Sophie Zabar, clairvoyante, cried out in pain at precisely the instant her twin brother, 500 km away, hit his thumb with a hammer. A skeptical scientist observed both events (brother's accident, Sophie's cry) from an airplane traveling at $\frac{12}{13}c$ to the right (see Fig. 12.19). Which event occurred first, according to the scientist? How *much* earlier was it, in seconds?



Problem 12.11 A record turntable of radius R rotates at angular velocity ω (Fig. 12.15). The circumference is presumably Lorentz-contracted, but the radius (being perpendicular to the velocity) is *not*. What's the ratio of the circumference to the diameter, in terms of ω and R? According to the rules of ordinary geometry, that has to be π. What's going on here? [This is known as **Ehrenfest's paradox**; for discussion and references see H. Arzelies, *Relativistic Kinematics*, Chap. IX (Elmsford, NY: Pergamon Press, 1966) and T. A. Weber, *Am. J. Phys.* **65**, 486 (1997).]



Problem 12.20

(a) Event A happens at point ($x_A = 5, y_A = 3, z_A = 0$) and at time t_A given by $ct_A = 15$; event B occurs at $(10, 8, 0)$ and $ct_B = 5$, both in system $\mathcal{S}$.

(i) What is the invariant interval between A and B?

(ii) Is there an inertial system in which they occur *simultaneously*? If so, find its velocity (magnitude and direction) relative to $\mathcal{S}$.

(iii) Is there an inertial system in which they occur at the same point? If so, find its velocity relative to $\mathcal{S}$

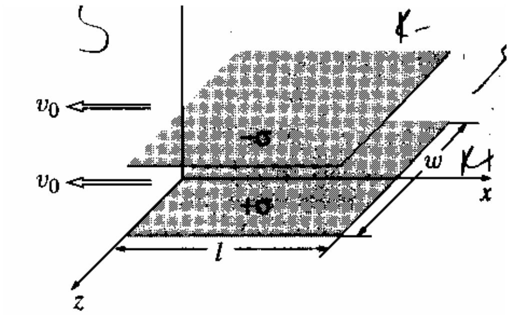
(b) Repeat part (a) for $A = (2, 0, 0)$, $ct = 1$; and $B = (5, 0, 0)$, $ct = 3$.

Problem 12.34 In the past, most experiments in particle physics involved stationary targets: one particle (usually a proton or an electron) was accelerated to a high energy E, and collided with a target particle at rest (Fig. 12.29a). Far higher *relative* energies are obtainable (with the same accelerator) if you accelerate *both* particles to energy E, and fire them at each other (Fig. 12.29b). *Classically,* the energy $\bar{E}$ of one particle, relative to the other, is just 4E (why?)—not much of a gain (only a factor of 4). But *relativistically* the gain can be *enormous*. Assuming the two particles have the same mass, m, show that

$$\bar{E} = \frac{2E^2}{mc^2} - mc^2. \tag{12.59}$$

EM HW 8b Thus. 24 May 2007 Griffiths **Ch.12** # 41, 43, 44

Problem 12.41 Why can't the electric field in Fig. 12.35b have a z component? After all, the *magnetic* field does.



Problem 12.43

(a) Check that Gauss's law, $\oint \mathbf{E} \cdot d\mathbf{a} = (1/\epsilon_0)Q_{\text{enc}}$, is obeyed by the field of a point charge in uniform motion, by integrating over a sphere of radius R centered on the charge.

(b) Find the Poynting vector for a point charge in uniform motion. (Say the charge is going in the z direction at speed v, and calculate $\mathbf{S}$ at the instant q passes the origin.)

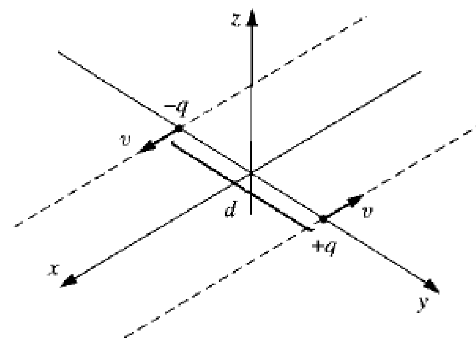
Problem 12.44

(a) Charge q_A is at rest at the origin in system S; charge q_B flies by at speed v on a trajectory parallel to the x axis, but at $y = d$. What is the electromagnetic force on q_B as it crosses the y axis?

(b) Now study the same problem from system $\bar{S}$, which moves to the right with speed v. What is the force on q_B when q_A passes the $\bar{y}$ axis? [Do it two ways: (i) by using your answer to (a) and transforming the force; (ii) by computing the fields in $\bar{S}$ and using the Lorentz force law.]

Problem 12.45 Two charges $\pm q$, are on parallel trajectories a distance d apart, moving with equal speeds v in opposite directions. We're interested in the force on $+q$ due to $-q$ at the instant they cross (Fig. 12.42). Fill in the following table, doing all the consistency checks you can think of as you go along.

	System A (Fig. 12.42)	System B ($+q$ at rest)	System C ($-q$ at rest)
$\mathbf{E}$ at $+q$ due to $-q$:			
$\mathbf{B}$ at $+q$ duc to $-q$:			
$\mathbf{F}$ on $+q$ due to $-q$:			



Problem 12.46

(a) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant.

(b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant.

(c) Suppose that in one inertial system $\mathbf{B} = 0$ but $\mathbf{E} \neq 0$ (at some point P). Is it possible to find another system in which the *electric* field is zero at P?

Problem 12.47 An electromagnetic plane wave of (angular) frequency ω is traveling in the x direction through the vacuum. It is polarized in the y direction, and the amplitude of the electric field is E_0.

(a) Write down the electric and magnetic fields, $\mathbf{E}(x, y, z, t)$ and $\mathbf{B}(x, y, z, t)$. [Be sure to define any auxiliary quantities you introduce, in terms of ω, E_0, and the constants of nature.]

(b) This same wave is observed from an inertial system $\bar{S}$ moving in the x direction with speed v relative to the original system S. Find the electric and magnetic fields in $\bar{S}$, and express them in terms of the $\bar{S}$ coordinates: $\bar{\mathbf{E}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$ and $\bar{\mathbf{B}}(\bar{x}, \bar{y}, \bar{z}, \bar{t})$. [Again, be sure to define any auxiliary quantities you introduce.]

(c) What is the frequency $\bar{\omega}$ of the wave in $\bar{S}$? Interpret this result. What is the wavelength $\bar{\lambda}$ of the wave in $\bar{S}$? From $\bar{\omega}$ and $\bar{\lambda}$, determine the speed of the waves in $\bar{S}$. Is it what you expected?