Physical Systems – spring 2007

EM HW 2a Tues.10 April 2007 Griffiths **Ch.7 #34, 42, 53.** Extra: 58 (a-c)

Problem 7.34 Suppose

$$\mathbf{E}(\mathbf{r}, t) = -\frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\theta(vt - r)\hat{\mathbf{r}}; \quad \mathbf{B}(\mathbf{r}, t) = 0$$

(the theta function is defined in Prob. 1.45b). Show that these fields satisfy all of Maxwell's equations, and determine ρ and $\mathbf{J}$. Describe the physical situation that gives rise to these fields.

(b) Let $\theta(x)$ be the **step function**:

$$\theta(x) \equiv \left\{ \begin{array}{ll} 1, & \text{if } x > 0 \\ 0, & \text{if } x \leq 0 \end{array} \right\}. \tag{1.95}$$

Show that $d\theta/dx = \delta(x)$.

Problem 7.42 In a **perfect conductor**, the conductivity is infinite, so $\mathbf{E} = 0$ (Eq. 7.3), and any net charge resides on the surface (just as it does for an *im*perfect conductor, in electro*statics*).

(a) Show that the magnetic field is constant ($\partial\mathbf{B}/\partial t = 0$), inside a perfect conductor.

(b) Show that the magnetic flux through a perfectly conducting loop is constant.

A **superconductor** is a perfect conductor with the *additional* property that the (constant) $\mathbf{B}$ inside is in fact *zero*. (This "flux exclusion" is known as the **Meissner effect.**[18])

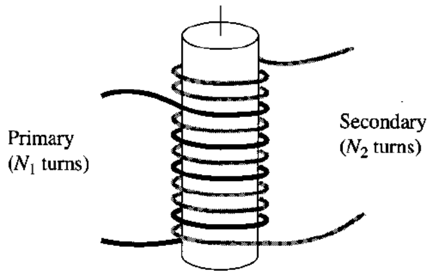
(c) Show that the current in a superconductor is confined to the surface.

(d) Superconductivity is lost above a certain critical temperature (T_c), which varies from one material to another. Suppose you had a sphere (radius a) above its critical temperature, and you held it in a uniform magnetic field $B_0\hat{\mathbf{z}}$ while cooling it below T_c. Find the induced surface current density $\mathbf{K}$, as a function of the polar angle θ.

Problem 7.53 Two coils are wrapped around a cylindrical form in such a way that the *same flux passes through every turn of both coils.* (In practice this is achieved by inserting an iron core through the cylinder; this has the effect of concentrating the flux.) The "primary" coil has N_1 turns and the secondary has N_2 (Fig. 7.54). If the current I in the primary is changing, show that the emf in the secondary is given by

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}, \tag{7.67}$$

where $\mathcal{E}_1$ is the (back) emf of the primary. [This is a primitive **transformer**—a device for raising or lowering the emf of an alternating current source. By choosing the appropriate number of turns, any desired secondary emf can be obtained. If you think this violates the conservation of energy, check out Prob. 7.54.]



Problem 7.54 A transformer (Prob. 7.53) takes an input AC voltage of amplitude V_1, and delivers an output voltage of amplitude V_2, which is determined by the turns ratio ($V_2/V_1 = N_2/N_1$). If $N_2 > N_1$ the output voltage is greater than the input voltage. Why doesn't this violate conservation of energy? *Answer:* Power is the product of voltage and current; evidently if the voltage goes *up*, the current must come *down*. The purpose of this problem is to see exactly how this works out, in a simplified model.

(a) In an ideal transformer the same flux passes through all turns of the primary and of the secondary. Show that in this case $M^2 = L_1 L_2$, where M is the mutual inductance of the coils, and L_1, L_2 are their individual self-inductances.

(b) Suppose the primary is driven with AC voltage $V_{in} = V_1 \cos(\omega t)$, and the secondary is connected to a resistor, R. Show that the two currents satisfy the relations

$$L_1 \frac{dI_1}{dt} + M\frac{dI_2}{dt} = V_1 \cos(\omega t); \quad L_2 \frac{dI_2}{dt} + M\frac{dI_1}{dt} = -I_2 R.$$

(c) Using the result in (a), solve these equations for $I_1(t)$ and $I_2(t)$. Assume I_1 has no DC component.

(d) Show that the output voltage ($V_{out}=I_2R$) divided by the input voltage (V_{in}) is equal to the turns ratio: $V_{in}/V_{out} = N_2/N_1$.

(e) Calculate the input power ($P_{in} = V_{in} I_1$) and the output power ($P_{out} = V_{out} I_2$) and show that their averages over a full cycle are equal.

EXTRA:

Problem 7.58 A certain transmission line is constructed from two thin metal "ribbons," of width w, a very small distance $h \ll w$ apart. The current travels down one strip and back along the other. In each case it spreads out uniformly over the surface of the ribbon.

(a) Find the capacitance per unit length, $\mathcal{C}$.

(b) Find the inductance per unit length, $\mathcal{L}$.

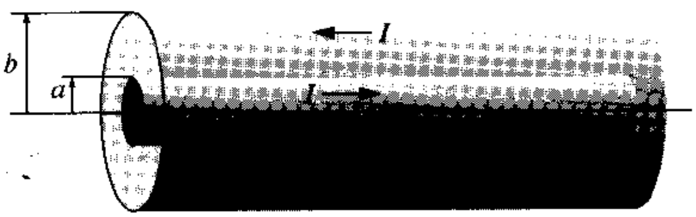
(c) What is the product $\mathcal{LC}$, numerically? [$\mathcal{L}$ and $\mathcal{C}$ will, of course, vary from one kind of transmission line to another, but their *product* is a universal constant—check, for example, the cable in Ex. 7.13—provided the space between the conductors is a vacuum. In the theory of transmission lines, this product is related to the speed with which a pulse propagates down the line: $v = 1/\sqrt{\mathcal{LC}}$.]

EM HW 2b Thus.12 April 2007 Griffiths **Ch.8 # 1**(a). Extra: #1(b)

Problem 8.1 Calculate the power (energy per unit time) transported down the cables of Ex. 7.13 and Prob. 7.58, assuming the two conductors are held at potential difference V, and carry current I (down one and back up the other).

Example 7.13

A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b) as shown in Fig. 7.39. Find the magnetic energy stored in a section of length l.



Solution: According to Ampère's law, the field between the cylinders is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi s}\hat{\boldsymbol{\phi}}.$$

Elsewhere, the field is zero. Thus, the energy per unit volume is

$$\frac{dW}{d\tau} = \frac{1}{2\mu_0}\left(\frac{\mu_0 I}{2\pi s}\right)^2 = \frac{\mu_0 I^2}{8\pi^2 s^2}.$$

The energy in a cylindrical shell of length l, radius s, and thickness ds, then, is $d\tau = 2\pi l s\, ds$

$$dW = \left(\frac{\mu_0 I^2}{8\pi^2 s^2}\right) 2\pi l s\, ds = \frac{\mu_0 I^2 l}{4\pi}\left(\frac{ds}{s}\right).$$

Integrating from a to b, we have:

$$W = \frac{\mu_0 I^2 l}{4\pi}\ln\left(\frac{b}{a}\right).$$

By the way, this suggests a very simple way to calculate the self-inductance of the cable. According to Eq. 7.29, the energy can also be written as $\frac{1}{2}LI^2$. Comparing the two expressions,[12]

$$L = \frac{\mu_0 l}{2\pi}\ln\left(\frac{b}{a}\right).$$

This method of calculating self-inductance is especially useful when the current is not confined to a single path, but spreads over some surface or volume. In such cases different parts of the current may circle different amounts of flux, and it can be very tricky to get L directly from Eq. 7.25.