

Problem 8.5 Consider an infinite parallel-plate capacitor, with the lower plate (at $z = -d/2$) carrying the charge density $-\sigma$, and the upper plate (at $z = +d/2$) carrying the charge density $+\sigma$.

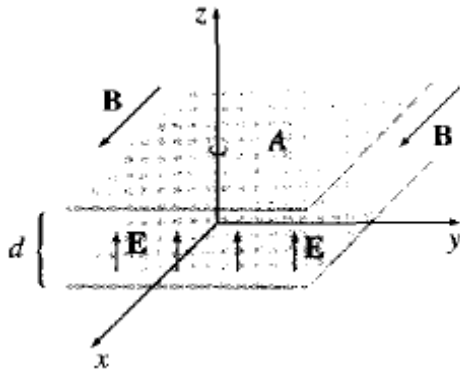
(a) Determine all nine elements of the stress tensor, in the region between the plates. Display your answer as a 3×3 matrix:

$$\begin{pmatrix} T_{xx} & T_{xy} & T_{xz} \\ T_{yx} & T_{yy} & T_{yz} \\ T_{zx} & T_{zy} & T_{zz} \end{pmatrix}$$

(b) Use Eq. 8.22 to determine the force per unit area on the top plate. Compare Eq. 2.51.

(c) What is the momentum per unit area, per unit time, crossing the xy plane (or any other plane parallel to that one, between the plates)?

(d) At the plates this momentum is absorbed, and the plates recoil (unless there is some nonelectrical force holding them in position). Find the recoil force per unit area on the top plate, and compare your answer to (b). [Note: This is not an *additional* force, but rather an alternative way of calculating the *same* force—in (b) we got it from the force law, and in (d) we did it by conservation of momentum.]



Problem 9.1 By explicit differentiation, check that the functions f_1 , f_2 , and f_3 in the text satisfy the wave equation. Show that f_4 and f_5 do *not*.

$$f_1(z, t) = Ae^{-b(z-vt)^2}, \quad f_2(z, t) = A \sin[b(z - vt)], \quad f_3(z, t) = \frac{A}{b(z - vt)^2 + 1}$$

all represent waves (with different shapes, of course), but

$$f_4(z, t) = Ae^{-b(bz^2 + vt)}, \quad \text{and} \quad f_5(z, t) = A \sin(bz) \cos(bvt)^3,$$

do *not*.

Problem 9.8 Equation 9.36 describes the most general **linearly** polarized wave on a string. Linear (or “plane”) polarization (so called because the displacement is parallel to a fixed vector \hat{n}) results from the combination of horizontally and vertically polarized waves of the *same phase* (Eq. 9.39). If the two components are of equal amplitude, but *out of phase* by 90° (say, $\delta_v = 0$, $\delta_h = 90^\circ$), the result is a *circularly* polarized wave. In that case:

(a) At a fixed point z , show that the string moves in a circle about the z axis. Does it go *clockwise* or *counterclockwise*, as you look down the axis toward the origin? How would you construct a wave circling the *other* way? (In optics, the clockwise case is called **right circular polarization**, and the counterclockwise, **left circular polarization**.)

(b) Sketch the string at time $t = 0$.

(c) How would you shake the string in order to produce a circularly polarized wave?

Accordingly, transverse waves occur in two independent states of **polarization**: you can shake the string up-and-down (“vertical” polarization—Fig. 9.8a),

$$\tilde{\mathbf{f}}_v(z, t) = \tilde{A}e^{i(kz - \omega t)} \hat{\mathbf{x}}, \quad (9.34)$$

or left-and-right (“horizontal” polarization—Fig. 9.8b),

$$\tilde{\mathbf{f}}_h(z, t) = \tilde{A}e^{i(kz - \omega t)} \hat{\mathbf{y}}, \quad (9.35)$$

or along any other direction in the xy plane (Fig. 9.8c):

$$\tilde{\mathbf{f}}(z, t) = \tilde{A}e^{i(kz - \omega t)} \hat{\mathbf{n}}. \quad (9.36)$$