

(a) Calculate I_r , in terms of dI_s/dt .

(b) The power ($I_r^2 R$) delivered to the ring must have come from the solenoid. Confirm this by calculating the Poynting vector just outside the solenoid (the *electric* field is due to the changing flux in the solenoid; the *magnetic* field is due to the current in the ring). Integrate over the entire surface of the solenoid, and check that you recover the correct total power.

Problem 8.10⁹ A sphere of radius R carries a uniform polarization \mathbf{P} and a uniform magnetization \mathbf{M} (not necessarily in the same direction). Find the electromagnetic momentum of this configuration. [Answer: $(4/9)\pi\mu_0 R^3 (\mathbf{M} \times \mathbf{P})$]

Problem 8.11¹⁰ Picture the electron as a uniformly charged spherical shell, with charge e and radius R , spinning at angular velocity ω .

(a) Calculate the total energy contained in the electromagnetic fields.

(b) Calculate the total angular momentum contained in the fields.

(c) According to the Einstein formula ($E = mc^2$), the energy in the fields should contribute to the mass of the electron. Lorentz and others speculated that the *entire* mass of the electron might be accounted for in this way: $U_{\text{em}} = m_e c^2$. Suppose, moreover, that the electron's spin angular momentum is entirely attributable to the electromagnetic fields: $L_{\text{em}} = \hbar/2$. On these two assumptions, determine the radius and angular velocity of the electron. What is their product, ωR ? Does this classical model make sense?

Problem 8.12¹¹ Suppose you had an electric charge q_e and a magnetic monopole q_m . The field of the electric charge is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{q_e}{r^2} \hat{\mathbf{r}},$$

of course, and the field of the magnetic monopole is

$$\mathbf{B} = \frac{\mu_0}{4\pi} \frac{q_m}{r^2} \hat{\mathbf{r}}.$$

Find the total angular momentum stored in the fields, if the two charges are separated by a distance d . [Answer: $(\mu_0/4\pi)q_e q_m$.]¹²

Problem 8.13 Paul DeYoung, of Hope College, points out that because the cylinders in Ex. 8.4 are left rotating (at angular velocities ω_a and ω_b , say), there is actually a residual magnetic field, and hence angular momentum in the fields, even after the current in the solenoid has been extinguished. If the cylinders are heavy, this correction will be negligible, but it is interesting to do the problem *without* making that assumption.

⁹For an interesting discussion and references, see R. H. Romer, *Am. J. Phys.* **63**, 777 (1995).

¹⁰See J. Higbie, *Am. J. Phys.* **56**, 378 (1988).

¹¹This system is known as **Thomson's dipole**. See I. Adawi, *Am. J. Phys.* **44**, 762 (1976) and *Phys. Rev. D* **31**, 3301 (1985), and K. R. Brownstein, *Am. J. Phys.* **57**, 420 (1989), for discussion and references.

¹²Note that this result is *independent of the separation distance* d (!); it points from q_e toward q_m . In quantum mechanics angular momentum comes in half-integer multiples of \hbar , so this result suggests that if magnetic monopoles exist, electric and magnetic charge must be quantized: $\mu_0 q_e q_m / 4\pi = n\hbar/2$, for $n = 1, 2, 3, \dots$, an idea first proposed by Dirac in 1931. If even *one* monopole exists somewhere in the universe, this would "explain" why electric charge comes in discrete units.