But it can be simplified by introducing the **Maxwell stress tensor**,

$$T_{ij} \equiv \epsilon_0 \left(E_i E_j - \frac{1}{2} \delta_{ij} E^2 \right) + \frac{1}{\mu_0} \left(B_i B_j - \frac{1}{2} \delta_{ij} B^2 \right). \tag{8.19}$$

are the same ($\delta_{xx} = \delta_{yy} = \delta_{zz} = 1$) and zero otherwise ($\delta_{xy} = \delta_{xz} = \delta_{yz} = 0$). Thus

$$T_{xx} = \frac{1}{2}\epsilon_0(E_x^2 - E_y^2 - E_z^2) + \frac{1}{2\mu_0}(B_x^2 - B_y^2 - B_z^2),$$

$$T_{xy} = \epsilon_0(E_x E_y) + \frac{1}{\mu_0}(B_x B_y),$$

$$\begin{aligned}(\nabla \cdot \overleftrightarrow{\mathbf{T}})_j &= \epsilon_0 \left[(\nabla \cdot \mathbf{E}) E_j + (\mathbf{E} \cdot \nabla) E_j - \frac{1}{2} \nabla_j E^2 \right] \\ &+ \frac{1}{\mu_0} \left[(\nabla \cdot \mathbf{B}) B_j + (\mathbf{B} \cdot \nabla) B_j - \frac{1}{2} \nabla_j B^2 \right]\end{aligned}$$

Thus the force per unit volume (Eq. 8.18) can be written in the much simpler form

$$\mathbf{f} = \nabla \cdot \overleftrightarrow{\mathbf{T}} - \epsilon_0 \mu_0 \frac{\partial \mathbf{S}}{\partial t}, \tag{8.21}$$

The *total* force on the charges in $\mathcal{V}$ (Eq. 8.15) is evidently

$$\mathbf{F} = \oint_{\mathcal{S}} \overleftrightarrow{\mathbf{T}} \cdot d\mathbf{a} - \epsilon_0 \mu_0 \frac{d}{dt} \int_{\mathcal{V}} \mathbf{S} \, d\tau. \tag{8.22}$$

Compare to

That argument applies to *any* surface charge; in the particular case of a conductor, the field is zero inside and $(\sigma/\epsilon_0)\hat{\mathbf{n}}$ outside (Eq. 2.48), so the average is $(\sigma/2\epsilon_0)\hat{\mathbf{n}}$, and the force per unit area is

$$\mathbf{f} = \frac{1}{2\epsilon_0}\sigma^2 \hat{\mathbf{n}}. \tag{2.51}$$

This amounts to an outward **electrostatic pressure** on the surface, tending to draw the conductor into the field, regardless of the sign of σ. Expressing the pressure in terms of the field just outside the surface,

$$P = \frac{\epsilon_0}{2} E^2. \tag{2.52}$$