QM 1b Physical Systems HW Thus.5.April 2007
Math review II: Ch.1.7 – 1.10
Do 1.**8.1** (p.45), 1.**8.3**, 1.**8.5**, 1.**9.2** (p.60), 1.**10.1**, 1.**10.2**

Exercise 1.8.1. (a) Find the eigenvalues and normalized eigenvectors of the matrix

$$\Omega = \begin{bmatrix} 1 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 4 \end{bmatrix}$$

(b) Is the matrix Hermitian? Are the eigenvectors orthogonal?

*Exercise 1.8.2.** Consider the matrix

$$\Omega = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

(a) Is it Hermitian?
(b) Find its eigenvalues and eigenvectors.
(c) Verify that $U^{\dagger}\Omega U$ is diagonal, U being the matrix of eigenvectors of Ω.

*Exercise 1.8.3.** Consider the Hermitian matrix

$$\Omega = \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$$

(a) Show that $\omega_1 = \omega_2 = 1;\ \omega_3 = 2$.
(b) Show that $|\omega = 2\rangle$ is any vector of the form

$$\frac{1}{(2a^2)^{1/2}}\begin{bmatrix} 0 \\ a \\ -a \end{bmatrix}$$

(c) Show that the $\omega = 1$ eigenspace contains all vectors of the form

$$\frac{1}{(b^2 + 2c^2)^{1/2}}\begin{bmatrix} b \\ c \\ c \end{bmatrix}$$

either by feeding $\omega = 1$ into the equations or by requiring that the $\omega = 1$ eigenspace be orthogonal to $|\omega = 2\rangle$.

*Exercise 1.9.2.** If H is a Hermitian operator, show that $U = e^{iH}$ is unitary. (Notice the analogy with c numbers: if θ is real, $u = e^{i\theta}$ is a number of unit modulus.)

*Exercise 1.10.1.** Show that $\delta(ax) = \delta(x)/|a|$. [Consider $\int \delta(ax)\, d(ax)$. Remember that $\delta(x) = \delta(-x)$.]

*Exercise 1.10.2.** Show that

$$\delta(f(x)) = \sum_i \frac{\delta(x_i - x)}{|df/dx_i|}$$

where x_i are the zeros of $f(x)$. Hint: Where does $\delta(f(x))$ blow up? Expand $f(x)$ near such points in a Taylor series, keeping the first nonzero term.