

Exercise 7.3.3. If $\psi(x)$ is even and $\phi(x)$ is odd under $x \rightarrow -x$, show that

$$\int_{-\infty}^{\infty} \psi(x)\phi(x) dx = 0$$

*Exercise 7.3.5.** Using the symmetry arguments from Exercise 7.3.3 show that $\langle n | X | n \rangle = \langle n | P | n \rangle = 0$ and thus that $\langle X^2 \rangle = (\Delta X)^2$ and $\langle P^2 \rangle = (\Delta P)^2$ in these states. Show that $\langle 1 | X^2 | 1 \rangle = 3\hbar/2m\omega$ and $\langle 1 | P^2 | 1 \rangle = \frac{3}{2}m\omega\hbar$. Show that $\psi_0(x)$ saturates the uncertainty bound $\Delta X \cdot \Delta P > \hbar/2$.

*Exercise 7.4.1.** Compute the matrix elements of X and P in the $|n\rangle$ basis and compare with the result from Exercise 7.3.4.

*Exercise 7.4.2.** Find $\langle X \rangle$, $\langle P \rangle$, $\langle X^2 \rangle$, $\langle P^2 \rangle$, $\Delta X \cdot \Delta P$ in the state $|n\rangle$.

Exercise 7.5.2. Project the relation

$$a|n\rangle = n^{1/2}|n-1\rangle$$

on the X basis and derive the recursion relation

$$H_n'(y) = 2nH_{n-1}(y)$$

using Eq. (7.3.22).

$$\begin{aligned} \psi_E(x) &= \psi_{(n+1/2)\hbar\omega}(x) = \psi_n(x) \\ &= \left(\frac{m\omega}{\pi\hbar 2^{2n}(n!)^2}\right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) H_n\left[\left(\frac{m\omega}{\hbar}\right)^{1/2} x\right] \end{aligned} \quad (7.3.22)$$

*Exercise 7.3.4.** Using Eqs. (7.3.23)–(7.3.25), show that

$$\langle n' | X | n \rangle = \left(\frac{\hbar}{2m\omega}\right)^{1/2} [\delta_{n',n+1}(n+1)^{1/2} + \delta_{n',n-1}n^{1/2}]$$

$$\langle n' | P | n \rangle = \left(\frac{m\omega\hbar}{2}\right)^{1/2} i[\delta_{n',n+1}(n+1)^{1/2} - \delta_{n',n-1}n^{1/2}]$$

*Exercise 7.4.5.** At $t = 0$ a particle starts out in $|\psi(0)\rangle = 1/2^{1/2}(|0\rangle + |1\rangle)$.

- (i) Find $|\psi(t)\rangle$; (ii) find $\langle X(0) \rangle = \langle \psi(0) | X | \psi(0) \rangle$, $\langle P(0) \rangle$, $\langle X(t) \rangle$, $\langle P(t) \rangle$;
- (iii) find $\langle \dot{X}(t) \rangle$ and $\langle \dot{P}(t) \rangle$ using Ehrenfest's theorem and solve for $\langle X(t) \rangle$ and $\langle P(t) \rangle$ and compare with part (ii).