

Exercise 7.4.2. Find $\langle X \rangle$, $\langle P \rangle$, $\langle X^2 \rangle$, $\langle P^2 \rangle$, $\Delta X \cdot \Delta P$ in the state $|n\rangle$.

|| Showed in 7.3.5 that $\langle X \rangle = 0$, $\langle P \rangle = 0$

$$\langle X^2 \rangle = \frac{\hbar}{2m\omega} (2n+1), \quad \langle P^2 \rangle = \frac{m\omega\hbar}{2} (2n+1)$$

$$\Delta X \Delta P =$$

Exercise 7.5.2. Project the relation

$$(1) \quad a|n\rangle = n^{1/2}|n-1\rangle$$

on the X basis and derive the recursion relation

$$H_n'(y) = 2nH_{n-1}(y)$$

using Eq. (7.3.22).

$$\begin{aligned} \psi_{n+1}(x) &= \psi_{(n+1/2)\hbar\omega}(x) = \psi_n(x) \\ &= \left(\frac{m\omega}{\pi\hbar 2^{2n}(n!)^2} \right)^{1/4} \exp\left(-\frac{m\omega x^2}{2\hbar}\right) H_n\left[\left(\frac{m\omega}{\hbar}\right)^{1/2} x\right] \end{aligned} \quad (7.3.22)$$

In the x -basis, (1) becomes

$$\langle n|n\rangle \rightarrow \int_{-\infty}^{\infty} \langle x|n\rangle \langle x|a|x'\rangle \langle x'|n\rangle dx' = \sqrt{n} \langle x|n-1\rangle. \quad \text{Use } a = \sqrt{\frac{m\omega}{2\hbar}} X + i\sqrt{\frac{\hbar}{2m\omega}} P$$

Matrix elements of a in the x -basis are

$$\begin{aligned} \langle x|a|x'\rangle &= \sqrt{\frac{m\omega}{2\hbar}} \langle x|X|x'\rangle + i\sqrt{\frac{\hbar}{2m\omega}} \langle x|P|x'\rangle \\ &= \sqrt{\frac{m\omega}{2\hbar}} x' \delta(x-x') + i\sqrt{\frac{\hbar}{2m\omega}} (-i\hbar) \delta(x-x') \frac{d}{dx} \end{aligned}$$

$$\begin{aligned} \text{So } \int_{-\infty}^{\infty} \langle x|a|x'\rangle \langle x'|n\rangle dx' &= \sqrt{\frac{m\omega}{2\hbar}} \int x' \delta(x-x') \psi_n(x') dx' + \hbar \sqrt{\frac{\hbar}{2m\omega}} \int \delta(x-x') \frac{d}{dx} \psi_n(x') dx \\ &= \sqrt{\frac{m\omega}{2\hbar}} x \psi_n(x) + \sqrt{\frac{\hbar}{2m\omega}} \frac{d}{dx} \psi_n(x) \end{aligned}$$