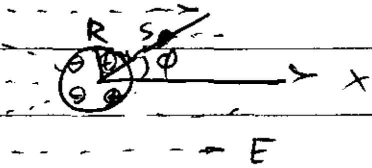
Problem 3.23 Solve Laplace's equation by separation of variables in *cylindrical* coordinates, assuming there is no dependence on z (cylindrical symmetry). [Make sure you find *all* solutions to the radial equation; in particular, your result must accommodate the case of an infinite line charge, for which (of course) we already know the answer.]

See attached worksheet. Solution:

$$V = \underbrace{C\ln s + D}_{k=0} + \sum_{k=1}^{\infty}\left(As^{k} + Bs^{-k}\right)e^{\pm ik\phi}$$

Problem 3.24 Find the potential outside an infinitely long metal pipe, of radius R, placed at right angles to an otherwise uniform electric field $\mathbf{E}_0$. Find the surface charge induced on the pipe. [Use your result from Prob. 3.23.]

This is much like Ex 3.8, p. 141



The conducting pipe is an equipotential

Far from the pipe, $V \to -E_0 X + C$

(i) $V(s \gg R) = -E_0 s\cos\phi$

(ii) We can set $V(s=R) = 0$

We must fit these boundary conditions to the solution of the Laplacian in cylindrical coordinates, above:

$$V = C\ln s + D + \sum_{k=1}^{\infty}\left(As^{k} + Bs^{-k}\right)\left(c\cos k\phi - d\sin k\phi\right)$$

C and D = 0 because V = 0 at S = R - no constant terms

d = 0 because of the orientation of the E field - no sines

And the only wave number is $k = 1$, since $V \sim \cos\phi$

so $V = \left(as + \frac{b}{s}\right)\cos\phi$ where I combined $a = \frac{A}{c}$, $b = Bc$

BC (ii) $V(s=R) = 0 = \left(aR + \frac{b}{R}\right)\cos\phi \to aR^2 = -b$

BC (i) $V(s \gg R) = \left(as + \frac{b}{\infty}\right)\cos\phi \simeq as\cos\phi = -E_0 s\cos\phi$

$a = -E_0 \to b = +E_0 R^2$

19.

6-19. In the early days of nuclear physics before the neutron was discovered, it was thought that the nucleus contained only electrons and protons. If we consider the nucleus to be a one-dimensional infinite well with $L = 10$ fm and ignore relativity compute the ground-state energy for (*a*) an electron and (*b*) a proton in the nucleus. (*c*) Compute the energy difference between the ground state and the first excited state for each particle. (Differences between energy levels in nuclei are found to be typically of the order of 1 MeV.)

We found that the energy level of an infinite square well is $E_n = \frac{n^2 h^2}{8mL^2}$ (problem 5.7 : $E = KE$ if $V=0$)

for $L = 10 \cdot 10^{-15}$ m, $a = \frac{h^2}{8L^2} = \frac{(1240 \text{ MeV} \cdot \text{fm})^2}{} =$

$E_n = \frac{n^2 a}{m}$

(a) For an electron, $E_{1_e} = \frac{a}{m_e = 0{,}511 \text{ MeV}} =$

(b) For a proton, $E_{1_p} = \frac{a}{m_p = 938 \text{ MeV}} =$

(c) $(E_2 - E_1)_{electron} = (2^2 - 1^2)E_{1_e} = (4-1)E_{1_e} = 3E_{1_e} =$ ______

$(E_2 - E_1)_{proton} = 3E_{1_p} =$ ______

27. The mass of the deuteron (the nucleus of the hydrogen isotope ^2H) is 1.88 GeV/c^2. How deep must a finite potential well be whose width is 2×10^{-15} m if there are two energy levels in the well?

$= 2a$

See MORE link referenced on p. 263

$$\frac{\sin ka}{\cos ka} = \tan ka = \frac{\alpha}{k} \qquad \textbf{6-41}$$

Substituting values of k and α from above, Equation 6-41 can also be written as

$$\tan\left(\frac{\sqrt{2mE}}{\hbar}a\right) = \sqrt{\frac{V_0 - E}{E}} \qquad \textbf{6-42}$$

Considering the odd solutions in the well, $\psi(x) = A_1 \sin kx$, an equivalent discussion leads to the condition that

$$-\cot ka = \frac{\alpha}{k} \qquad \textbf{6-43}$$

Though tedious to solve analytically, the solutions to these transcendental equations can be readily found graphically. The solutions are those points where the graphs of

notice that this figure is missing from the text...

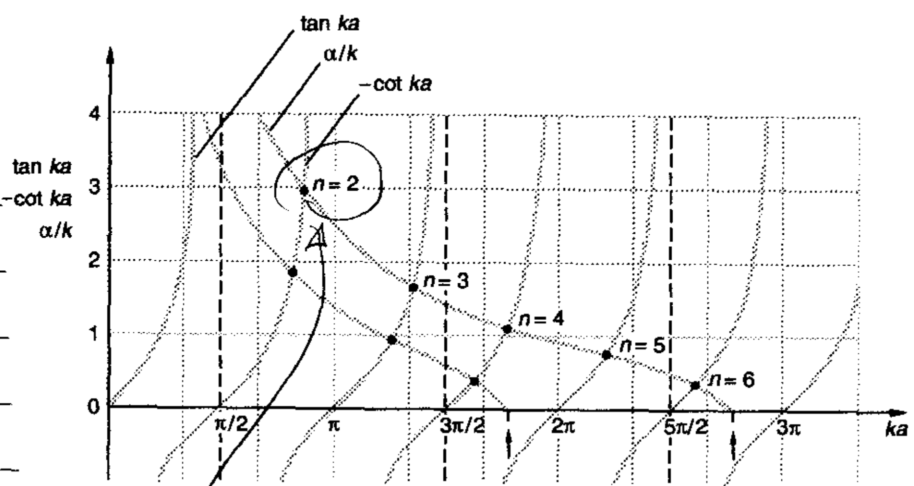


Fig. 6-14 Graphical solutions of Equations 6-41 and 6-43. Two different curves of α/k are shown, each corresponding to a different value of V_0. The value of V_0 in each case is given by the value of ka where $\alpha/k = 0$, indicated by the small arrows. For example, the top α/k curve has $\alpha/k = 0$ for $ka = 2.75\pi$, or $(2mV_0)^{1/2}a/\hbar = 2.75\pi$. Allowed values of E are those given by the values of ka at the intersections of the α/k and $\tan ka$ and α/k and $-\cot ka$ curves.

(Continued)

There will be at least two energy levels in the well if $ka > \frac{\pi}{2}$ where $ka = a\sqrt{2mE}/\hbar \rightarrow$ solve for $E = E_2$