Physical Systems

fall EM HW #2 - Ch 3 - Griffiths - solutions - Zita

14 Oct 02

Ch 3 # $\frac{2}{115}$, $\frac{3}{116}$, [$\frac{5}{121}$, (image $\frac{9}{126}$ + $\frac{2.22}{82}$) line of charge] next week

Problem 3.2 In one sentence, justify **Earnshaw's Theorem:** *a charged particle cannot be held in stable equilibrium by electrostatic forces alone.* As an example, consider the cubical lattice of fixed charges in Fig. 3.4. It *looks*, offhand, as though a positive charge at the center would be suspended in midair—repelled away from each corner. Where is the leak in this "electrostatic bottle"? (To harness nuclear fusion as a practical energy source it is necessary to heat a plasma (soup of charged particles) to fantastic temperatures—so hot that contact would vaporize any ordinary pot. Earnshaw's Theorem says that electrostatic containment is also out of the question. Fortunately, it *is* possible to confine a hot plasma *magnetically*.)

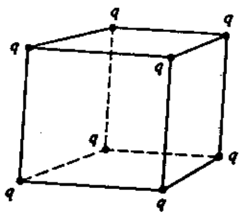


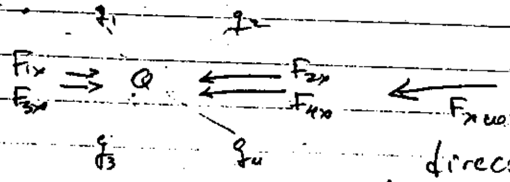
Figure 3.4

Stable equilibria occur at potential energy minima ($W \sim qV$), but solutions of $\nabla^2 V = 0$ (V) can have no local minimum, therefore any electrostatic equilibrium must be unstable.

($\nabla \cdot E = 0$)

$\nabla \cdot (\nabla V) = \nabla^2 V = 0$

For example, at the center point above, all forces cancel — it is a point of equilibrium. But this equilibrium is unstable - a small displacement (of a test charge) from the center of the cube results in a force imbalance.



$F_{x\,net}$ is in the direction of the displacement ∴ the displacement grows, out the sides of box

(V cannot have a minimum at center of cube - it must be some sort of saddle point)

UNSTABLE equilibrium