Physical Systems Midterm – E&M and Modern - Tues.6.Feb.2007

This is a CLOSED-BOOK exam to be taken in class. This is designed as a half-hour quiz on each topic, one hour total. You have 2 hours to do it.
SHOW YOUR WORK neatly, and include units where appropriate, to receive full credit.
Please circle or underline your answers for clarity.
Express answers in *simplest exact form* whenever possible.
Order-of-magnitude estimates are usually fine for numerical problems.

8 E(r)
14 Mod
14
36

(please sign legibly) ZITA SOLUTIONS

I affirm that I have worked this exam with WITHOUT using a calculator, text, HW, quizzes, computer, classmates, or other resources, except equations given to us by faculty.

4 **1. (a) Find the electric field** inside a sphere which carries a charge density proportional to the distance from the origin, ρ = c r, for some constant c. [*Hints:* This charge density is *not* uniform, and you must integrate to get the enclosed charge q(r<R). A spherical volume element is dτ = r² sinθ dr dθ dϕ where (0<θ<π) and (0<ϕ< 2π).]

$\int d\phi = 2\pi \quad \int \sin\theta\, d\theta = -\cos\theta\Big|_0^\pi = -(-1-1) = 2$

~ **(b) Set up your calculations for finding the electrostatic potential everywhere.** Be specific and complete. You need not actually calculate V(r)

2 (c) How could you find the energy in this charge configuration? You need not calculate it, but set it up. (Use the back of this page if you like.)

1 (a) $\int d\tau = \int r^2 dr \int \sin\theta\, d\theta \int d\phi = \underline{4\pi \int r^2 dr}$ and $\rho = \frac{dq}{d\tau}$ so

$q(r) = \int \rho\, d\tau = 4\pi \int r^2 \rho\, dr = 4\pi \int r^2 (cr)\, dr = 4\pi c \int_0^r r^3 dr = \frac{4\pi c}{4} r^4$

1 $\underline{q(r<R) = \pi c r^4}$ inside the sphere. $Q_{tot} = q(R) = \underline{\pi c R^4}$

1 $\oint \vec{E} \cdot d\vec{a} = \frac{q(r)}{\epsilon_0} = E \cdot 4\pi r^2 \rightarrow E(r) = \frac{q(r)}{4\pi\epsilon_0 r^2}$

1 INSIDE : $E(r<R) = \frac{\pi c r^4}{4\pi\epsilon_0 r^2} = \frac{c r^2}{4\epsilon_0}$

OUTSIDE : $E(r>R) = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{\pi c R^4}{4\pi\epsilon_0 r^2} = \frac{cR^4}{4\epsilon_0 r^2}$

2 (b) $V(r>R) = -\int_{r=\infty}^{r} \vec{E}_{out} \cdot d\vec{l} = -\int \frac{cR^4}{4\epsilon_0 r^2} dr = \frac{-cR^4}{4\epsilon_0} \int r^{-2} dr = \frac{-cR^4}{4\epsilon_0} \frac{r^{-1}}{-1}\Big|_\infty^r$
OUTSIDE

$V(r>R) = \frac{cR^4}{4\epsilon_0}\left(\frac{1}{r} - \frac{1}{\infty}\right) = \frac{cR^4}{4\epsilon_0 r}$ and $V(R) = \frac{cR^3}{4\epsilon_0}$

INSIDE $V(r<R) = V(R) - \int_{r=R}^{r} E_{in} \cdot dl = \frac{cR^3}{4\epsilon_0} - \frac{c}{4\epsilon_0}\int_R^r r^2 dr = \frac{cR^3}{4\epsilon_0} - \frac{c}{4\epsilon_0} \frac{r^3}{3}\Big|_R^r$

Continued

INSIDE $V(r<R) = \frac{CR^3}{4\epsilon_0} - \frac{C}{4\epsilon_0}\left.\frac{r^3}{3}\right|_R^r$

$$= \frac{CR^3}{4\epsilon_0} - \frac{C}{4\epsilon_0}\left(\frac{r^3}{3} - \frac{R^3}{3}\right)$$

$$= \frac{C}{4\epsilon_0}\left[R^3 + \frac{R^3}{3} - \frac{r^3}{3}\right]$$

$$= \frac{C}{4\epsilon_0}\left[\frac{4R^3}{3} - \frac{r^3}{3}\right]$$

$$V(r<R) = \frac{C}{3\epsilon_0}\left[R^3 - \frac{r^3}{4}\right]$$

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2 @ $\frac{\text{ENERGY}}{\text{volume}} = \frac{1}{2\epsilon_0}E^2 = \frac{1}{2}\rho V$

$$\text{Energy} = \frac{1}{2\epsilon_0}\int E^2\, d\tau = \frac{1}{2}\int \rho V\, d\tau$$

all space $\qquad$ $(r<R)$ where there is charge

Modern Physics: Ch.1-5

3 **2. In relativity, what is gamma γ?**

(a) Give its defining equation. $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

(b) Explain its role in length contraction and time dilation. Say an object has proper length L_o and lifetime τ_o in its rest frame. If it moves with speed v relative to an observer at rest, the observer will measure shorter $L = \frac{L_o}{\gamma}$ and longer $\tau = \gamma\tau_o$.

(c) Is gamma greater or less than one?

If $v=0$ then $\gamma = \frac{1}{\sqrt{1-0}} = 1$. If $v>0$ then $\gamma = \frac{1}{\sqrt{1-\text{something}}} = \frac{1}{<1} > 1$

7 **3. Consider two inertial reference frames, possibly moving with respect to each other. When an observer in each frame measures the following quantities, which measurements made by the two observers MUST yield the same results?** EXPLAIN your reason for each answer.

(a) The distance between two events - Not the same - length contraction

(b) The value of the mass of a proton - Not the same - apparent or measured mass depends on motion!

(c) The speed of light - always the same - fundamental postulate

(d) The time interval between two events - not the same - time dilation

(e) Newton's second law - not the same; $F = \gamma\, dp/dt$ depends on motion

(f) The order of the elements in the periodic table - unchanged number of nuclei

(g) The value of the electron charge - unchanged (though E fields not invariant)

4 **4. The total energy of a particle is three times its rest energy.**
(a) Find u/c for the particle.
(b) Find its momentum.

$E^2 = (mc^2)^2 + (pc)^2$

$E = \gamma mc^2 = 3mc^2$

(a) $\gamma = 3 = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}} \rightarrow 1-\frac{u^2}{c^2} = \frac{1}{9}$

$1-\frac{1}{9} = \frac{u^2}{c^2} = \frac{9}{9}-\frac{1}{9} = \frac{8}{9} \rightarrow \frac{u}{c} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$

(b) $pc = \sqrt{E^2-(mc^2)^2} = \sqrt{(3mc^2)^2-(mc^2)^2}$

$= mc^2\sqrt{9-1} = \sqrt{8}\, mc^2$

$pc = 2\sqrt{2}\, mc^2 \qquad p = \underline{2\sqrt{2}\, mc}$

or $p = \gamma mu = 3m\left(\frac{2\sqrt{2}}{3}c\right) = 2\sqrt{2}\, mc$

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5. Gravitational redshift: (a) If a transmission of frequency f_0 from Earth is measured by a GPS satellite in orbit above Earth, does the satellite measure a higher or lower frequency? Explain: Satellite measures a lower frequency. The radiation loses energy to climb out of the gravitational potential well of Earth.

(b) By approximately what fraction is the frequency shifted? A factor of:

i.	two	iv.	a million
ii.	ten	v.	a billion
iii.	1000	vi.	other

We calculated this with $\frac{\Delta f}{f} = \frac{gh}{c^2}$

4 **6. Photoelectric effect:** Consider an experiment with a constant stopping potential on a fixed plate (constant work function).

(a) If the intensity of the light is increased, how will the photo-ejected electron beam change? Why? There will be more electrons of the same energy.

(b) If the color of the light is turned bluer, how will the photo-ejected electron beam change? Why?

Faster electrons, because $E_{in} = E_{out}$

Not more electrons! $hf = \phi + \frac{1}{2}mv^2$

light energy — electron kinetic energy

7. Bohr model for the hydrogen atom: (a) Write an equation for each assumption in this model:

2 (i) An electron orbits a proton due to their electrostatic interaction.

$F = ma \rightarrow \frac{ke^2}{r^2} = \frac{mv^2}{r}$

Virial Theorem $\left(E_{tot} = KE - PE = \frac{PE}{2} = -\frac{ke^2}{2r}\right)$

3 (ii) An integer number (n) of electron deBroglie wavelengths fit in each orbit.

$n\lambda = 2\pi r, \quad E = \frac{hc}{\lambda} = pc \rightarrow \frac{h}{\lambda} = p = mv \rightarrow \lambda = \frac{h}{mv} = \frac{2\pi r}{n}$

$\lambda = \frac{2\pi r}{n}$

(b) Write each of your equations above as $v^2(r)$:

(i) $v^2 = \frac{ke^2}{mr}$ (ii) $v^2 = \left(\frac{hn}{2\pi r m}\right)^2 = \left(\frac{n\hbar}{rm}\right)^2$

(c) Solve for r(n):

$\frac{1}{7}$

$\frac{mr}{ke^2} = \frac{r^2m^2}{n^2\hbar^2} \rightarrow r = \frac{n^2\hbar^2}{ke^2m}$

Extra credit: Combine your r(n) with ~~(i)~~ the virial theorem to derive E(n). Use the back if you like.

$E = \frac{-ke^2}{2r} = \frac{-ke^2}{2}\left(\frac{ke^2m}{n^2\hbar^2}\right) = -\frac{k^2e^4m}{2n^2\hbar^2}$

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Equations given for EM/modern midterm:

$$\oint \bar{E} \cdot d\bar{a} = \frac{q}{\epsilon_0} \qquad \rho = \frac{dq}{d\tau}$$

$$\int \bar{E} \cdot d\bar{l} = -V, \quad \bar{\nabla} V = -\bar{E}$$

$$d\tau = \int_0^r r^2 dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} \phi \, d\phi$$

$$\frac{dW}{d\tau} = \text{energy density} = \frac{E^2}{2\epsilon_0} = \frac{\rho V}{2}$$