3.3 General solution to $\nabla^2 V = 0$ for $V(r)$ and $V(s)$

116

(a) Spherical coordinates:

(1.73) $$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi}$$

42

(b) Cylindrical coordinates:

(1.82) $$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

44

3.3 General solution to $\nabla^2 V = 0$ for $V(r)$ and $V(s)$

116

ⓐ Spherical coordinates:

(1.73) 42
$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial V}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 V}{\partial \phi}$$

$$\frac{\partial V(r)}{\partial \theta} = 0 \qquad \frac{\partial V(r)}{\partial \phi} = 0$$

$$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\underbrace{\left(r^2 \frac{\partial V(r)}{\partial r}\right)}_{\text{constant}} = 0$$

constant: $r^2 \frac{\partial V}{\partial r} = C$

$$V = \int \partial V = \int \frac{C}{r^2} dr = \underline{-\frac{C}{r} + k = V(r)}$$

Spherical

ⓑ Cylindrical coordinates:

(1.82) 44
$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\left(s \frac{\partial V}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

$$\frac{\partial V(s)}{\partial \phi} = 0 \qquad \frac{\partial V(s)}{\partial z} = 0$$

$$\nabla^2 V = \frac{1}{s}\frac{\partial}{\partial s}\underbrace{\left(s \frac{\partial V(s)}{\partial s}\right)}_{\text{constant}} = 0$$

constant: $s \frac{\partial V}{\partial s} = C$

$$V = \int dV = \int \frac{C}{s} ds = \underline{C \ln s + k = V}$$

cylindrical