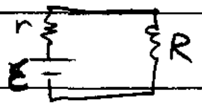
Ch 7 HW - Electrodynamics #2, 5, 7, 10, 11, 12, 17, 25

HINTS & SELECTED ANSWERS

#2 / 290 — RC circuit - See lecture notes

#5 / 293



$R_{tot} = r + R$. Find $I = \frac{\mathcal{E}}{R_{tot}}$ and $P = I^2 R$. Minimize P: $\frac{dP}{dR} = 0$

#7 / 299 — motional emf - see lecture notes $v = v_0 e^{-\frac{B^2 l^2}{mR} t}$ from $F = ma = m\frac{dv}{dt}$ | $W = \frac{1}{2} m v_0^2$ from $\frac{dW}{dt} = I^2 R$

#10 / 300 — generator - See lecture notes

#11 / 300 — Falling loop: Draw force diagram, find $F_B = IlB = \frac{B^2 l^2 v}{R}$. Balance with $F_g = mg$ to find v

$v_{terminal} = 1.2 \frac{cm}{s}$, $v(t) = v_t (1 - e^{-ct})$ find c from $\Sigma F = 0$

#12 / 305 — $\Phi = A_{\text{little loop}} B$, $\mathcal{E} = -\frac{d\Phi}{dt}$, $I = \frac{\mathcal{E}}{R} = \frac{\pi a^2 \omega}{4R} B_0 \sin \omega t$

#17 / 309 — Solenoid field is nonzero only inside the solenoid, so $\Phi = B_{solenoid} A_{solenoid}$

$\mathcal{E} = -\frac{d\Phi}{dt} = IR \rightarrow I = \frac{\pi a^2 \mu_0 n k}{R}$

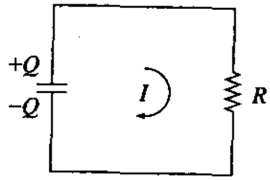
#25 / 316 — LC circuit: See lecture notes: $\Sigma V = 0$, $I = \frac{dQ}{dt} \rightarrow$ solve $\frac{d^2 Q}{dt^2} = -\frac{Q}{LC}$ for $Q(t)$ and $I(t) = -Q\omega \sin \omega t$

$C = \frac{Q}{V} = \frac{Q_0}{V_0} = \frac{Q(t)}{V(t)}$

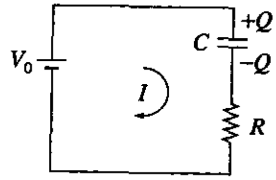
Problem 7.2 A capacitor C has been charged up to potential V_0; at time $t = 0$ it is connected to a resistor R, and begins to discharge (Fig. 7.5a).

(a) Determine the charge on the capacitor as a function of time, $Q(t)$. What is the current

We learned in mechanics that $V_c = \frac{Q}{C}$ and $V_R = IR$



(a)



(b)

$\Sigma V = 0 = V_c + V_R$

$$\frac{Q}{C} = -IR = -R\frac{dQ}{dt} \rightarrow \frac{dQ}{dt} = -\frac{1}{RC}Q = -\frac{Q}{\tau} \rightarrow Q = Q_0 e^{-t/\tau}, \quad \tau = RC$$

$$Q = CV_0 e^{-t/RC}$$

$$I = \frac{dQ}{dt} = -\frac{CV_0}{\tau}e^{-t/RC} = +\frac{V_0}{R}e^{-t/RC}$$

(b) What was the original energy stored in the capacitor (Eq. 2.55)? By integrating Eq. 7.7, confirm that the heat delivered to the resistor is equal to the energy lost by the capacitor.

Now imagine *charging up* the capacitor, by connecting it (and the resistor) to a battery of fixed voltage V_0, at time $t = 0$ (Fig. 7.5b).

(c) Again, determine $Q(t)$ and $I(t)$.

(d) Find the total energy output of the battery ($\int V_0 I\,dt$). Determine the heat delivered to the resistor. What is the final energy stored in the capacitor? What fraction of the work done by the battery shows up as energy in the capacitor? [Notice that the answer is independent of R!]

Energy $= \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{1}{2}CV_0^2$ stored in capacitor $= W_c$

Heat lost to resistor? Power $= \frac{dW}{dt} = I^2R \rightarrow W = \int I^2R\,dt = -\left(\frac{V_0}{R}\right)^2 R \int e^{-2t/\tau}\,d\tau$

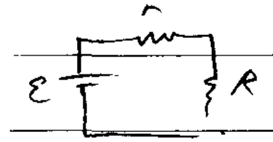
$$W_R = -\frac{V_0^2}{R}\left(\frac{-\tau}{2}\right)e^{-t/\tau}\Big|_0^\infty = \frac{V_0^2\tau}{2R} = \frac{V_0^2 RC}{2R} = \frac{1}{2}V_0^2 C \ \checkmark$$

Energy supplied by battery $= \int V_0 I\,dt = V_0 \int \frac{V_0}{R} e^{-t/\tau}\,dt$

$$W_b = \frac{V_0^2}{R}(-\tau)e^{-t/\tau}\Big|_0^\infty = \frac{V_0^2 RC}{R} = V_0^2 C$$

$W_{cap} = W_{resistor}$ and $W_b = W_{cap} + W_{resistor}$

Problem 7.5 A battery of emf $\mathcal{E}$ and internal resistance r is hooked up to a variable "load" resistance, R. If you want to deliver the maximum possible power to the load, what resistance R should you choose? (You can't change $\mathcal{E}$ and r, of course.)



From hints, $R_{tot} = r + R$ and $I = \frac{\mathcal{E}}{r+R}$ so $P = IV = I(IR) = I^2R = \frac{\mathcal{E}^2 R}{(r+R)^2}$

Maximize $P(R)$: $\frac{dP}{dR} = \mathcal{E}^2 \frac{d}{dR}\left[\frac{R}{(r+R)^2}\right] = \mathcal{E}^2\left[\frac{1}{(r+R)^2} - \frac{2R}{(r+R)^3}\right]$

$$= \mathcal{E}^2\left[\frac{r+R-2R}{(r+R)^3}\right] = \mathcal{E}^2\left(\frac{r-R}{(r+R)^3}\right)$$

$\frac{dP}{dR} = 0$ when $r = R$: Maximum power through balanced load.

Check: Max or Min? $\frac{d^2P}{dR^2} = \mathcal{E}^2\left[\frac{-1}{(r+R)^3} - \frac{3(r-R)}{(r+R)^4}\right] = \mathcal{E}^2\left[\frac{-(r+R)-3r+3R}{(r+R)^3}\right]$

P $\cap$ $\cup$ $\to R$ $\quad \left.\frac{d^2P}{dR^2}\right|_{r=R} = \mathcal{E}^2\left(\frac{-2R}{(2R)^3}\right) < 0 \therefore$ MAX

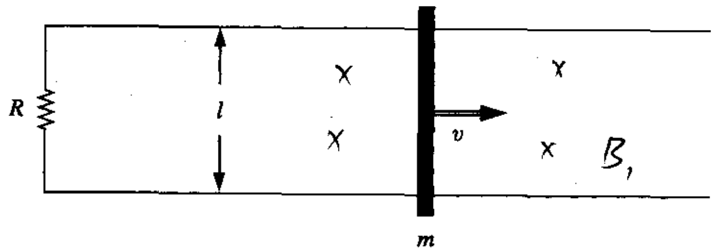
Problem 7.10 A square loop (side a) is mounted on a vertical shaft and rotated at angular **velocity** ω (Fig. 7.18). A uniform magnetic field **B** points to the right. Find the $\mathcal{E}(t)$ for this **alternating current** generator.

We showed in class that the flux changes as the loop turns: $\Phi = \vec{B}\cdot\vec{A} = BA\cos\theta = Ba^2\cos\omega t$

so $\frac{d\phi}{dt} = -Ba^2\omega\sin\omega t = -\mathcal{E}$

$$Ba^2\omega\sin\omega t = \mathcal{E}$$

Problem 7.7 A metal bar of mass m slides frictionlessly on two parallel conducting rails a distance l apart (Fig. 7.16). A resistor R is connected across the rails and a uniform magnetic field **B**, pointing into the page, fills the entire region.



We showed in class that you can find the motional emf $\mathcal{E} = E \cdot l$ from either the Lorentz force: $F = qvB = qE \rightarrow E = vB$

$\mathcal{E} = Blv$ or from Faraday's law: $\mathcal{E} = \frac{d\Phi}{dt} = B\frac{dA}{dt} = Blv$ ✓

(a) If the bar moves to the right at speed v, what is the current in the resistor? In what direction does it flow?

(b) What is the magnetic force on the bar? In what direction?

(c) If the bar starts out with speed v_0 at time $t = 0$, and is left to slide, what is its speed at a later time t?

(a) $\mathcal{E} = Blv$ and $\mathcal{E} = IR$ so $I = \frac{Blv}{R}$

The direction of I is to oppose the increase in flux. $\boxed{B_2 \odot \circlearrowleft I}$

Counterclockwise I will drive B_2 opposite B_1

(b) Magnetic force $F_m = q\bar{v} \times B = Il \times B = \frac{B^2 l^2 v}{R}$ $\quad \xleftarrow{F_m}$ opposing the motion

(c) $F = ma = m\frac{dv}{dt} = -\frac{B^2 l^2}{R} v$ Familiar separable diffeq

$\frac{dv}{v} = -\frac{B^2 l^2}{Rm} dt = \frac{-dt}{\tau}$ where $\tau = \frac{Rm}{B^2 l^2}$

$\ln v = -\frac{t}{\tau} + k$

$V(t) = V_0 e^{-t/\tau}$

7.7 (d) : (d) The initial kinetic energy of the bar was, of course, $\frac{1}{2}mv_0^2$. Check that the energy delivered to the resistor is exactly $\frac{1}{2}mv_0^2$.

$$\text{Power} = \frac{\text{Energy}}{\text{time}} = \frac{dW}{dt} = \frac{d}{dt}\int F \cdot dx = F \cdot v \quad \text{So} \quad W = \int F \cdot v\, dt$$

We found $F(t) = \frac{B^2 l^2 v(t)}{R}$ and $v(t) = V_0 e^{-t/\tau}$, $\tau = \frac{Rm}{B^2 l^2}$

$$W = \frac{B^2 l^2}{R}\int v^2 dt = \frac{m}{\tau} V_0^2 \int_0^\infty e^{-2t/\tau} dt = \frac{m}{\tau} V_0^2 \left(-\frac{\tau}{2}\right) e^{-2t/\tau}\Big|_0^\infty$$

$$W = \frac{1}{2} m V_0^2 \checkmark$$

Another way: $P = \frac{dW}{dt} = I^2 R = \frac{B^2 l^2 v^2}{R^2} R \rightarrow W = \frac{B^2 l^2}{R}\int v^2 dt = \text{SAME}$

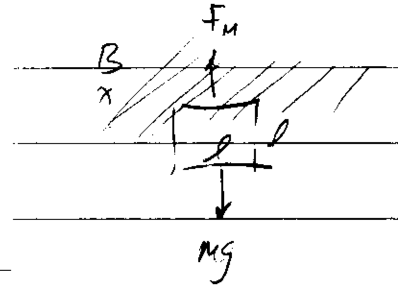
Problem 7.12 A long solenoid, of radius a, is driven by an alternating current, so that the field inside is sinusoidal: $\mathbf{B}(t) = B_0 \cos(\omega t)\,\hat{\mathbf{z}}$. A circular loop of wire, of radius $a/2$ and resistance R, is placed inside the solenoid, and coaxial with it. Find the current induced in the loop, as a function of time.

$$\text{Area} = \pi r^2 = \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}, \quad \frac{dB}{dt} = -\omega B_0 \sin \omega t$$

$$\frac{d\Phi}{dt} = A\frac{dB}{dt} = -\frac{\pi a^2}{4}\omega B_0 \sin\omega t = -\mathcal{E} = -IR$$

Current induced in the loop $I(t) = \frac{\mathcal{E}}{R} = \frac{\pi a^2 \omega B_0 \sin \omega t}{4R}$

Problem 7.11 A square loop is cut out of a thick sheet of aluminum. It is then placed so that the **top** portion is in a uniform magnetic field **B**, and allowed to fall under gravity (Fig. 7.19). (In the diagram, shading indicates the field region; **B** points into the page.) If the magnetic field is 1 T (a pretty standard laboratory field), find the terminal velocity of the loop (in m/s). Find the velocity of the loop as a function of time. How long does it take (in seconds) to reach, say, 90% of the terminal velocity? What would happen if you cut a tiny slit in the ring, breaking the circuit? [*Note:* The dimensions of the loop cancel out; determine the actual *numbers*, in the units indicated.]

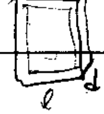


A magnetic force opposes the fall because an emf is induced:

$\mathcal{E} = Blv$ as we found in #7.7, so $F_M(t) = \dfrac{B^2 l^2 v(t)}{R}$.

Terminal velocity: no more acceleration: $\Sigma F = 0$ so $F_M = mg$

$\rightarrow V_{term} = \dfrac{mgR}{B^2 l^2}$



mass = density · volume

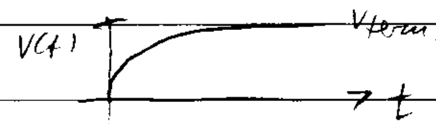
$m = \rho(4l \cdot d^2)$

Resistance = resistivity · $\dfrac{\text{length}}{\text{cross-sectional area}}$

$R = \dfrac{1}{\sigma}\dfrac{4l}{d^2}$ (σ = conductivity)

$V_{term} = \dfrac{(\rho 4 l d^2) g \left(\frac{1}{\sigma}\frac{4l}{d^2}\right)}{B^2 l^2} = \dfrac{16\rho g}{B^2 \sigma}$ $\quad \rho = 2.7\times10^3\ \text{kg/m}^3$, $\sigma = \dfrac{1}{2.8\times10^{-8}\ \Omega m}$

$= \dfrac{16\,(2.7\times10^3 \frac{kg}{m^3})(10\frac{m}{s^2})}{1T^2\left(\frac{1}{2.8\times10^{-8}\Omega m}\right)}$



$V_{term} = 1.2\times10^{-2}\ \frac{m}{s}$

We can solve for v as a function of time the same as we did in #7.7, except $V(0)=0$ now: $F = m\frac{dv}{dt} = -m\frac{v}{\tau} \rightarrow \ln v = -\frac{t}{\tau} + k$

$V(t) = C_1 e^{-t/\tau} + C_2$ $\qquad V(t) = C_1(e^{-t/\tau} - 1)$ $\qquad \tau = \dfrac{Rm}{B^2 l^2} = \dfrac{V_t}{g}$

$V(0) = C_1 e^0 + C_2$ $\qquad V(\infty) = C_1(e^{-\infty} - 1)$ $\qquad \tau = 1.2\times10^{-3}\,s$

$0 = C_1 + C_2 \rightarrow C_1 = -C_2$ $\qquad V_{term} = C_1(0-1) \rightarrow C_1 = -V_{term}$

Finally, $V(t) = V_{term}(1 - e^{-t/\tau})$. Solve for t:

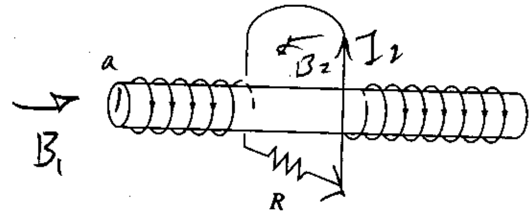
$1 - \dfrac{V(t)}{V_{term}} = e^{-t/\tau} \rightarrow t = -\tau \ln\left(1 - \dfrac{V(t)}{V_{term}}\right) = -\tau\ln(1-0.9) = \underline{2.8\ ms.}$

Problem 7.17 A long solenoid of radius a, carrying n turns per unit length, is looped by a wire with resistance R, as shown in Fig. 7.27.

$n = \frac{N}{\ell}$

(a) If the current in the solenoid is increasing at a constant rate ($dI/dt = k$), what current flows in the loop, and which way (left or right) does it pass through the resistor?

(b) If the current I in the solenoid is constant but the solenoid is pulled out of the loop, turned around, and reinserted, what total charge passes through the resistor?



Solenoid's current creates a magnetic field of strength

$\oint B \cdot dl = N\mu_0 I \rightarrow B = \frac{N\mu_0 I}{\ell} = n\mu_0 I$

The field exists only inside the area $A = \pi a^2$.

So the flux $\Phi = BA = \pi a^2 n \mu_0 I$ and the Emf induced in any loop around the solenoid (regardless of size) is

$$\mathcal{E} = \frac{-d\Phi}{dt} = -\pi a^2 n \mu_0 \frac{dI}{dt} = -k\pi a^2 n \mu_0$$

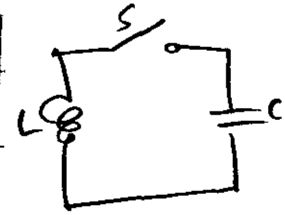
The current induced in the outer wire can be found from

$$\mathcal{E} = I_2 R \rightarrow I_2 = \frac{\mathcal{E}}{R} = \frac{-k\pi a^2 n \mu_0}{R}$$

As expected from Lenz's law, this current is in the opposite direction, and the field B_2 which it creates opposes the increase in flux due to the growing I_1 (growing B_1)

(b) If I_1 = constant then flux through the outer loop decreases when the solenoid is removed (current flows one way) and increases when solenoid is put back (current flows the other way) So the total charge $+Q - Q = 0$.

Problem 7.25 A capacitor C is charged up to a potential V and connected to an inductor L, as shown schematically in Fig. 7.38. At time $t = 0$ the switch S is closed. Find the current in the circuit as a function of time. How does your answer change if a resistor R is included in series with C and L?

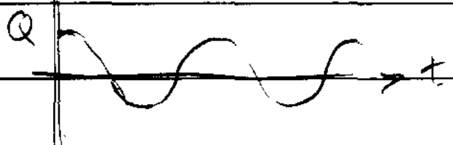


$\sum V = 0 = V_C + V_L = \frac{Q}{C} + L\frac{dI}{dt}$ (we did this in mechanics too)

$\frac{1}{C}Q = -L\frac{d^2Q}{dt^2}$ Guess $Q = Q_0 e^{i\omega t}$. Then $\ddot{Q} = -\omega^2 Q$

$= +L\omega^2 Q$

$\frac{1}{LC} = \omega^2 \rightarrow$ $\underline{Q = Q_0 \cos \omega t}$ (Since $Q(0) = Q_0 = CV$)



$\underline{I = \frac{dQ}{dt} = -\omega Q_0 \sin \omega t}$

If we add a resistor then $\sum V = V_L + V_R + V_C = L\frac{dI}{dt} + IR + \frac{Q}{C}$

$0 = L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q$

This is exactly like the damped harmonic oscillator (undriven) which we solved in mechanics: $\sum F = m\ddot{x} = -kx - b\dot{x} = 0$ (CM lecture notes 24 Oct 02 EJZ)

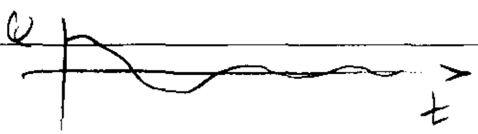
$\omega_m^2 = \frac{k}{m}$, $2\gamma_m = \frac{b}{m}$ $\qquad \ddot{x} = -\omega_m^2 x - 2\gamma_m \dot{x}$

$\omega_L^2 = \frac{1}{LC}$, $2\gamma_L = \frac{R}{L}$ $\qquad \ddot{Q} = -\frac{1}{LC}Q - \frac{R}{L}\dot{Q}$

The mechanical oscillator had solutions $x = e^{-\gamma t}(A_+ e^{qt} + A_- e^{-qt})$ where $q = i\omega_d$ and the damped frequency $\omega_d = \sqrt{\omega_m^2 - \gamma_m^2}$

So the electrical oscillator has solutions $Q = e^{-\gamma_L t}(A_+ e^{pt} + A_- e^{-pt})$ where $p = i\omega_{Ld}$ and $\omega_{Ld} = \sqrt{\omega_L^2 - \gamma_L^2}$

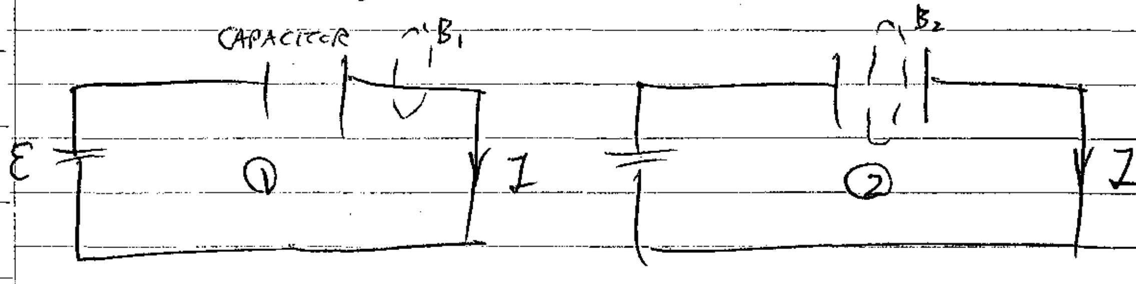
$= \sqrt{\frac{1}{LC} - \frac{R}{2L}}$



BOTTOM LINE: OSCILLATIONS DAMP

Giancoli p. 788
Griffiths p. 324

Find the missing term in Ampere's Law:



Amperian loop #1

$$\oint \bar{B}_1 \cdot d\bar{l} = \mu_0 I$$

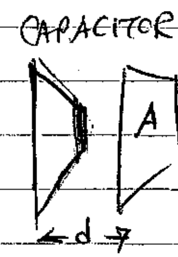
Amperian loop #2

$$\oint \bar{B}_2 \cdot d\bar{l} = 0 \;!?$$

$B_2 = B_1$, but what is its source?

The electric field between the capacitor plates changes in ②

$Q = CV \qquad C = \epsilon_0 \frac{A}{d} \qquad V = Ed$



$Q =$ ______

$$I_D = \frac{dQ}{dt} = \_\_\_\_\_\_ \frac{dE}{dt} = \epsilon_0 \frac{d\Phi_E}{dt}$$

This "displacement current", caused by the changing E, accounts for the magnetic field BETWEEN the capacitor plates

$$\oint B \cdot dl = \mu_0 (I + I_D) = \_\_\_\_\_\_\_\_\_\_\_\_$$

anywhere

In differential form, $\bar{\nabla} \times \bar{B} = \mu_0 (J + \_\_\_\_\_\_\_\_\_\_)$

Ch 9. EM Waves
Giancoli Ch 32

Changing magnetic flux → Emf. Consider Φ due to a current I

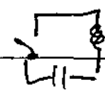
$\Phi = \underset{\uparrow}{L} I$ $\qquad\qquad \mathcal{E} = -\frac{d\Phi}{dt} =$ ______

constant of proportionality $\qquad\qquad$ back-emf (voltage) due to inductor L

$L = \frac{\Phi}{I}$

Ex: RL circuit $\Sigma V = IR + L\frac{dI}{dt} = 0$ 

RC circuit $\Sigma V = \frac{dQ}{dt}R + \frac{Q}{C} = 0$ 

LC circuit $\Sigma V = \frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$ 

	electric fields	magnetic fields
Energy	$U_E = \frac{1}{2}CV^2$	$U_B = \frac{1}{2}LI^2$
energy/vol	$u_E = \frac{1}{2}\epsilon_0 E^2$	$u_B = \frac{1}{2\mu_0}B^2$

MAXWELL EQNS

Gauss	$\oint \bar{E}\cdot d\bar{A} = q/\epsilon_0$	(i) $\bar{\nabla}\cdot\bar{E} = \rho/\epsilon_0$	$\rho \equiv \frac{\text{charge}}{\text{volume}}$
	$\oint \bar{B}\cdot d\bar{A} = 0$	(ii) $\bar{\nabla}\cdot\bar{B} = 0$	
Faraday	$\oint \bar{E}\cdot d\bar{l} = -\frac{d\Phi_B}{dt}$	(iii) $\bar{\nabla}\times\bar{E} = -\frac{\partial\bar{B}}{\partial t}$	$\Phi_B \equiv \int \bar{B}\cdot d\bar{A}$
Ampere	$\oint \bar{B}\cdot d\bar{l} = \mu_0\epsilon_0\frac{d\Phi_E}{dt} + \mu_0 I$	(iv) $\bar{\nabla}\times\bar{B} = \mu_0\epsilon_0\frac{\partial\bar{E}}{\partial t} + \mu_0\bar{J}$	$\bar{J} = \frac{\text{current}}{\text{area}}$

33 BC: HE tan : $\Delta H = \frac{B_1^{\parallel}}{\mu_1} - \frac{B_2^{\parallel}}{\mu_2} = \bar{K}\times\hat{n}$, $\quad \Delta E = E_1^{\parallel} - E_2^{\parallel} = 0$

BD perp : $\Delta D = \epsilon_1 E_1^{\perp} - \epsilon_2 E_2^{\perp} = \sigma$, $\quad \Delta B = B_1^{\perp} - B_2^{\perp} = 0$

discontinuities due to surface charge or current $\qquad$ CONTINUOUS

Ch 7 - worksheets - wk 9

- DERIVE THE ENERGY STORED IN INDUCTOR = work done against back-emf

317 Power $= \frac{dW}{dt}$ Power $= -\mathcal{E} I$ $\Phi = LI$, $-\mathcal{E} = \frac{d\Phi}{dt}$

$= \_\_\_\_\_\_$

Integrate $\int dW = W = \int \_\_\_\_\_\_ \, dt =$

☐ Do details of p. 318 to derive $W = \frac{1}{2\mu_0}\int B^2 \, d\tau \rightarrow u_B = \frac{1}{2\mu_0} B^2$

- DERIVE EM WAVES FROM MAXWELL EQNS (in vacuum: $J=0$) 375

(a) $\nabla \times$ (iii) Faraday : use vector identity - front cover

VI (ii) :

$\frac{\partial}{\partial t}$ (iv) :

$\nabla^2 E = \_\_\_\_\_\_ \frac{\partial^2 E}{\partial t^2}$

(b) $\nabla \times$ (iv) Ampere : use vector identity

V.I. (ii) :

$\frac{\partial}{\partial t}$ (iii) :

$\nabla^2 B = \_\_\_\_\_\_ \frac{\partial^2 B}{\partial t^2}$

$\mu_0 = 4\pi \times 10^{-7} \left(\frac{Tm}{A} = \frac{N}{A^2}\right)$ WAVES : $\frac{\partial^2 f}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

$\epsilon_0 = 8.85 \times 10^{-12} \left(\frac{C^2}{Nm^2}\right)$ SPEED $v = \_\_\_\_\_\_$

WORKSHEET

Characteristics of EM radiation: Assume PLANE

Monochromatic waves (single, constant wavelength λ and frequency ω)

376 traveling in z-direction: $\tilde{E}(z,t) = \tilde{E}_0 e^{i(kz-\omega t)}$, $\tilde{B}(z,t) = \tilde{B}_0 e^{i(kz-\omega t)}$

constant (complex) amplitudes

$\bar{\nabla}\cdot\bar{E} =$ $= 0$ in vacuum

$\bar{\nabla}\cdot\bar{B} =$ $= 0$ in vacuum

- What component of E & B vanishes?

$\therefore$ TRANSVERSE WAVES: perpendicular to direction of travel.

$\bar{\nabla}\times\bar{E} =$

$\frac{\partial\bar{B}}{\partial t} =$

- How are x & y components of E & B related to each other?

$\therefore$ MUTUALLY PERPENDICULAR

380 In general, $\tilde{\bar{E}}(\bar{r},t) = \tilde{E}_0 e^{i(\bar{k}\cdot\bar{r}-\omega t)} \hat{n}$

379 $\tilde{\bar{B}}(\bar{r},t) = \tilde{B}_0 e^{i(\bar{k}\cdot\bar{r}-\omega t)} (\hat{k}\times\hat{n}) = \frac{1}{c}\hat{k}\times\bar{E}$

- Do Prob 9.9 (380)

$$\nabla \cdot \tilde{E} = \frac{\partial}{\partial x} E_x + \frac{\partial}{\partial y} E_y + \frac{\partial}{\partial z} E_z = E_{0_z} ik e^{i(kz-\omega t)} = 0$$

$$(\tilde{E}_0)_z = (\tilde{B}_0)_z = 0$$

$$\nabla \times E = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \frac{\partial}{\partial z} \\ E_{0x} e^{\sim} & E_{0y} e^{\sim} & 0 \end{vmatrix} = \begin{matrix} \hat{i} \left(-\frac{\partial}{\partial z} E_{0y} e^{i(kz-\omega t)} \right) \\ - \hat{j} \left(-\frac{\partial}{\partial z} E_{0x} e^{i(kz-\omega t)} \right) \end{matrix}$$

$$= -\hat{i} E_{0y} ik e^{\sim} + \hat{j} E_{0x} ik e^{\sim}$$

$$\frac{\partial B}{\partial t} = -i\omega (B_{0x} \hat{i} + B_{0y} \hat{j}) e^{\sim}$$

$$-k (\tilde{E}_0)_y = \omega (\tilde{B}_0)_x \qquad k (\tilde{E}_0)_x = \omega (\tilde{B}_0)_y$$

$$\tilde{B}_0 = \frac{k}{\omega} (\hat{z} \times \tilde{E}_0)$$

Energy of light $U = \overset{\substack{\text{momentum}\\ \downarrow}}{p}c = \frac{hc}{\lambda} = hf$ where $\lambda = \frac{2\pi}{k}$, $\omega = 2\pi f$

h = Planck const $= 6.63 \times 10^{-34}$ J·s

Speed: $v = f\lambda = \frac{\omega}{k} = c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$ (Dispersion: $v = \frac{d\omega}{dk}$)

energy/vol $u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2 = \epsilon_0 E^2 = \frac{1}{\mu_0} B^2$ since $\left(\frac{1}{2}\epsilon_0 E^2 = \frac{1}{2\mu_0} B^2\right)$

Energy flux density $= \frac{\text{Power}}{\text{Area}} = \frac{dU/dt}{\text{Area}} = \frac{\frac{dU}{dx}\frac{dx}{dt}}{\text{Area}} = \frac{dU}{d\,\text{vol}} \cdot c$

$$S = c \cdot u = c\epsilon_0 E^2 = \frac{c}{\mu_0} B^2 = \frac{EB}{\mu_0}$$ Giancoli: 801

$$\bar{S} = \frac{1}{\mu_0} \bar{E} \times \bar{B}$$ (Time averaged $\langle \bar{S} \rangle = \frac{1}{2}|\bar{S}|$; Intensity $= \frac{1}{2} c\epsilon_0 E^2$)

• P = Radiation pressure $= \frac{\text{Force}}{\text{area}} = \frac{dp/dt}{\text{area}}$

$\frac{dp}{dt} = \frac{1}{c}\, dU/dt$ absorption

$\frac{dp}{dt} = \frac{2}{c}\, dU/dt$ reflection

$\langle S \rangle = \frac{\frac{1}{2}\, dU/dt}{\text{area}}$

9.10 / 382 Find P for absorption & reflection.

due Mon. 9. Dec. 02

Final E&M homework:

(a) Griffiths Ch 7 # $\underset{320}{27}$, 29, 42, 53 (rec: 31, 48, 59, $\frac{5.21}{234}$, $\frac{7.35}{328}$)

(b) Griffiths Ch 9 # 1, 9, 10, and complete worksheets from lecture.

(c) Giancoli Ch 29 # $\underset{755}{61}$, #44, Ch 32 # $\underset{809}{47}$, 49

Hints and answers:

(a) 27: $\frac{\mu_0}{4\pi} \, u^2 I^2 h \ln\left(\frac{b}{a}\right)$

29: $\frac{\mathcal{E}_0}{R} e^{-t/\tau}$

42: (a) use Faraday in differential form

(b) use Faraday in integral form

(c) use Ampere (d) use Eqn (5.68) to find $-\frac{3}{2}\frac{B_0}{\mu_0}\sin\theta\,\hat{\phi}$

(b) 9.9 (a) Hint: $\bar{k} = -\frac{\omega}{c}\hat{x}$, $\hat{n} = \hat{z}$

(b) $\bar{k} = \frac{\omega}{c}\frac{1}{\sqrt{3}}(\hat{x}+\hat{y}+\hat{z})$, $\hat{n} = \frac{\hat{x}-\hat{z}}{\sqrt{2}}$. Calculate $\bar{k}\cdot\bar{r}$ and $\bar{k}\times\hat{n}$

to find $\frac{E_0}{c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z)-\omega t\right]\left(\frac{-\hat{x}+2\hat{y}-\hat{z}}{\sqrt{6}}\right)$

9.10: $4.3\times10^{-6}\,\frac{N}{m^2}$, 8.3×10^{-11} atm

Problem 7.42 In a **perfect conductor**, the conductivity is infinite, so $\mathbf{E} = 0$ (Eq. 7.3), and any net charge resides on the surface (just as it does for an *imperfect* conductor, in electro*statics*).

(a) Show that the magnetic field is constant ($\partial \mathbf{B}/\partial t = 0$), inside a perfect conductor.

(b) Show that the magnetic flux through a perfectly conducting loop is constant.

(a) Faraday's law: $\frac{\partial \bar{B}}{\partial t} = -\bar{\nabla} \times \bar{E}$: if $\bar{E} = 0$ then $\bar{\nabla} \times \bar{E} = 0$ so $\frac{\partial B}{\partial t} = 0 \rightarrow B =$ constant inside perfect conductor.

(b) Faraday's law: $\oint \bar{E} \cdot d\bar{l} = -\frac{d\phi}{dt} = 0$ if $\bar{E} = 0$, so $\Phi =$ constant through perfectly conducting loop.

A **superconductor** is a perfect conductor with the *additional* property that the (constant) **B** inside is in fact *zero*. (This "flux exclusion" is known as the **Meissner effect**.[18])

(c) Show that the current in a superconductor is confined to the surface.

Ampere's law: $\oint \bar{B} \cdot d\bar{l} = \mu_0 I_{enclosed}$. If there were current inside the conductor, then you could draw an amperian loop INSIDE which enclosed nonzero I, and B_{in} would be nonzero.

Since $B_{in} = 0 \rightarrow I_{in} = 0$.

[Even if you use $\bar{\nabla} \times \bar{B} = \mu_0 \bar{J} + \mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t}$, $\bar{E} = 0 = \bar{B}$ so $\bar{J}_{inside} = 0$]

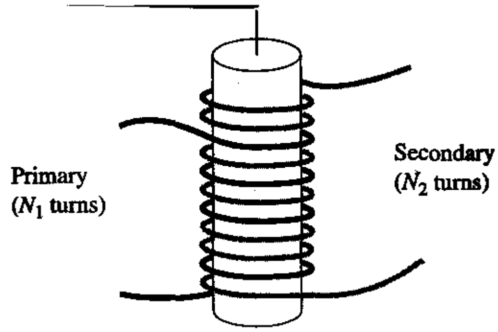
NO CURRENT INSIDE $\rightarrow$ ANY CURRENT MUST BE ON SURFACE

(d) Superconductivity is lost above a certain critical temperature (T_c), which varies from one material to another. Suppose you had a sphere (radius a) above its critical temperature, and you held it in a uniform magnetic field $B_0\hat{z}$ while cooling it below T_c. Find the induced surface current density **K**, as a function of the polar angle θ.

Problem 7.53 Two coils are wrapped around a cylindrical form in such a way that the *same flux passes through every turn of both coils*. (In practice this is achieved by inserting an iron core through the cylinder; this has the effect of concentrating the flux.) The "primary" coil has N_1 turns and the secondary has N_2 (Fig. 7.54). If the current I in the primary is changing, show that the emf in the secondary is given by

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1}, \tag{7.67}$$

where $\mathcal{E}_1$ is the (back) emf of the primary. [This is a primitive **transformer**—a device for raising or lowering the emf of an alternating current source. By choosing the appropriate number of turns, any desired secondary emf can be obtained. If you think this violates the conservation of energy, check out Prob. 7.54.]



Flux through one loop of primary = Flux through one loop of secondary

$$\Phi_1/N_1 = \Phi = \Phi = \Phi_2/N_2$$

$$\mathcal{E}_1 = -N_1 \frac{d\Phi}{dt} \quad \text{and} \quad \mathcal{E}_2 = -N_2 \frac{d\Phi}{dt}$$

$$\frac{d\Phi}{dt} = \frac{d\Phi}{dt}$$

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2} \longrightarrow \frac{\mathcal{E}_1}{\mathcal{E}_2} = \frac{N_1}{N_2} \checkmark$$

WAVE EQN : $\frac{\partial^2 f}{\partial z^2} = v^2 \frac{\partial^2 f}{\partial t^2}$

Problem 9.1 By explicit differentiation, check that the functions f_1, f_2, and f_3 in the text satisfy the wave equation. Show that f_4 and f_5 do ***not***.

$$f_1(z,t) = Ae^{-b(z-vt)^2}, \quad f_2(z,t) = A\sin[b(z-vt)], \quad f_3(z,t) = \frac{A}{b(z-vt)^2+1}$$

all represent waves (with different shapes, of course), but

$$f_4(z,t) = Ae^{-b(bz^2+vt)}, \quad \text{and} \quad f_5(z,t) = A\sin(bz)\cos(bvt)^3,$$

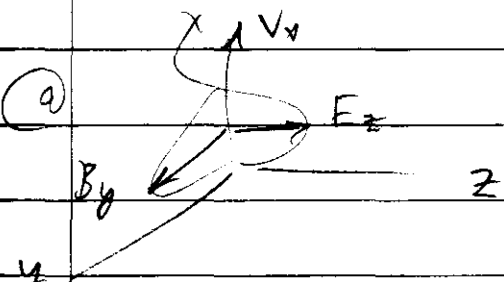
do *not*.

$$\mathbf{E}(\mathbf{r}, t) = E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta)\, \hat{\mathbf{n}}, \tag{9.51}$$

$$\mathbf{B}(\mathbf{r}, t) = \frac{1}{c} E_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t + \delta)(\hat{\mathbf{k}} \times \hat{\mathbf{n}}). \tag{9.52}$$

Problem 9.9 Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is (a) traveling in the negative x direction and polarized in the z direction; (b) traveling in the direction from the origin to the point (1, 1, 1), with polarization parallel to the $x\,z$ plane. In each case, sketch the wave, and give the explicit Cartesian components of $\mathbf{k}$ and $\hat{\mathbf{n}}$.

(a)



$\vec{k} = -\frac{\omega}{c}\hat{x}$, $\hat{n} = \hat{z}$

$\bar{k}\cdot\bar{r} = (-\frac{\omega}{c}\hat{x})\cdot(x\hat{x} + y\hat{y} + z\hat{z})$

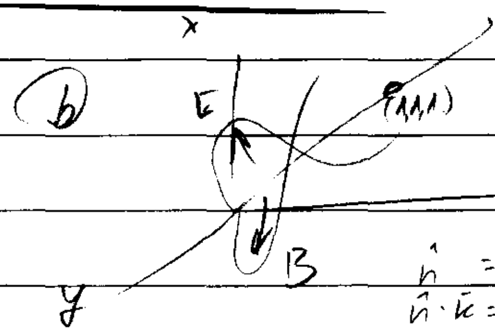
$= -\frac{\omega}{c}x$

$\hat{k}\times\hat{n} = -\hat{x}\times\hat{z} = \hat{y}$

$\vec{E} = E_0 \cos\left(\frac{\omega}{c}x + \omega t\right)\hat{z}$

$\bar{B} = \frac{E_0}{c}\cos\left(\frac{\omega}{c}x + \omega t\right)\hat{y}$

(b)



$\hat{n} = \alpha\hat{x} + \beta\hat{z}$

$\hat{n}\cdot k = 0 \rightarrow \beta = -\alpha = -\frac{1}{\sqrt{2}}$

$\bar{k} = \frac{\omega}{c}\left(\frac{\hat{x}+\hat{y}+\hat{z}}{\sqrt{3}}\right)$ $\hat{n} = \frac{\hat{x}-\hat{z}}{\sqrt{2}}$

$\bar{k}\cdot\bar{r} = \frac{\omega}{\sqrt{3}c}(x+y+z)$

$\hat{k}\times\hat{n} = \frac{1}{\sqrt{6}}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ 1 & 1 & 1\\ 1 & 0 & -1\end{vmatrix} = \frac{1}{\sqrt{6}}(-\hat{x}+2\hat{y}-\hat{z})$

$\bar{E} = E_0\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{\hat{x}-\hat{z}}{\sqrt{2}}\right)$

$\bar{B} = \frac{E_0}{c}\cos\left[\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right]\left(\frac{-\hat{x}+2\hat{y}-\hat{z}}{\sqrt{6}}\right)$

EMT HW#12 due Wed 30 Nov. 94 - Electrodynamics - ZTA
Physics 335 Ch 7 #5, 9, 12, 16, 22 (also discuss ratio of currents) 25, 28 (justify concisely) (XC: 29)

6

Problem 7.5

(a) Show that electrostatic forces alone cannot be used to drive current around a circuit. (In fact, *within the source* current flows in the direction *opposite* to **E**.)

(b) A rectangular loop of wire is situated so that one end is between the plates of a parallel-plate capacitor (Fig. 7.10), oriented parallel to the field $E = (\sigma/\epsilon_0)$. The other end is way outside, where the field is essentially zero. If the width of the loop is h and its total resistance is R, what current flows? Explain.

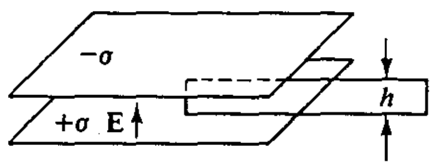
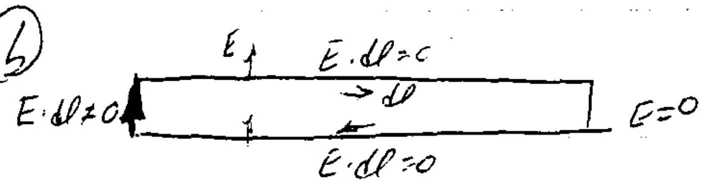


Figure 7.10

(a) In electrostatics, $\nabla \times \vec{E} = 0$, so Stokes' Theorem says $\oint \vec{E} \cdot d\vec{l} = 0$

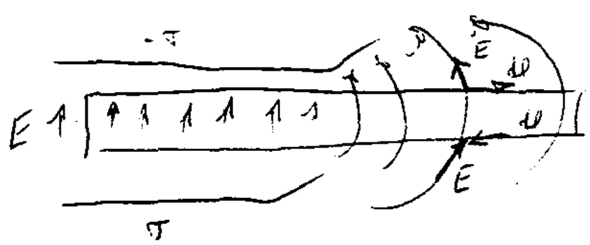
Work done around a circuit $W = \oint \vec{F} \cdot d\vec{l} = \int q\vec{E} \cdot d\vec{l} \to 0$

(b)



looks like $\oint E \cdot dl \neq 0$?

But we neglected FRINGING FIELDS:



← $E \cdot dl < 0$ on top loop

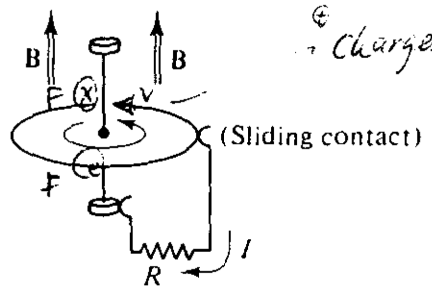
← $E \cdot dl < 0$ on bottom loop

$E \cdot \vec{l} > 0$ inside - must be exactly canceled by negative $E \cdot dl$ due to fringing fields along top and bottom $\to I = 0$

Problem 7.9 Suppose the disc in Fig. 7.15 is rotating at an angular velocity ω, and its radius is a. What is the emf of the circuit, and what current flows through the resistor?

Example 4

A metal disc rotates about a vertical axis, through a uniform field **B**, pointing up. A circuit is made by connecting one end of a resistor to the axle and the other end to a sliding contact which touches the outer edge of the disc (Fig. 7.15). Current will flow in the direction indicated, yet the flux through the cir-



∴ charges moving with v feel outward $F = q v B = q \omega r B$ all over the plate

Figure 7.15

cuit does not seem to be changing. I'll let you calculate the emf (Problem 7.9), using the Lorentz force law. (The trouble with the flux rule is that it assumes the current flows along a well-defined path, whereas in this example the current spreads out over the whole disc. When you stop to think about it, it's not even clear what the "flux through the circuit" *means* in this context.)

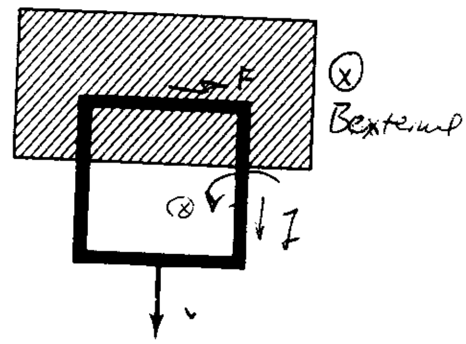
$$\mathcal{E} = \int \frac{F}{q} \cdot dl = \int_{r=0}^{a} B\omega r \, dr = \frac{B\omega a^2}{2}$$

$$\mathcal{E} = IR \rightarrow I = \frac{\mathcal{E}}{R} = \frac{B\omega a^2}{2R}$$

Problem 7.12 ^{11} A square loop is cut out of a thick sheet of aluminum. It is then placed so that the top portion is in a uniform magnetic field **B**, and allowed to fall under gravity (Fig. 7.19). (In the diagram, shading indicates the field region; **B** points into the page.) If the magnetic field is 1 T (a pretty standard laboratory field), find the terminal velocity of the loop (in m/s). Find the velocity of the loop as a function of time. How long does it take (in seconds) to reach, say, 90% of the terminal velocity? What would happen if you cut a tiny slit in the ring, breaking the circuit? (*Note:* The dimensions of the loop cancel out; determine the actual numbers, in the units indicated.)

$\vec{F} = q\vec{v}\times\vec{B} \rightarrow$ clockwise current due to motional Emf

Decreasing $\otimes$ flux $\rightarrow I$ drives B_{in} to oppose $\frac{d\Phi}{dt}$ ✓ (clockwise)



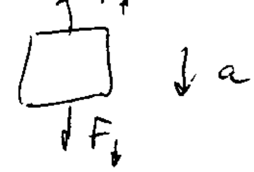
Now consider interaction between this induced current and the external B:

$F \uparrow$, $\longrightarrow I$, $\otimes$ B_{ext} $\qquad F = I\,l\times B \longrightarrow$ UPWARD force opposes fall

Qualitative stuff above, quantitative stuff below:

$\mathcal{E} = Blv = IR \rightarrow I = \frac{Blv}{R}$

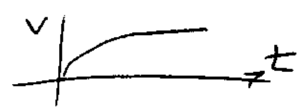
$F_\uparrow = IlB = \frac{vl^2B^2}{R}$ opposes $F_\downarrow = mg$



$ma = \Sigma F$

(1) $m\frac{dv}{dt} = mg - \frac{vl^2B^2}{R}$ has solutions (2) $v(t) = V_m(1-e^{-t/\tau})$

$v(0)=0, \quad v(\infty) = V_m$



What are terminal velocity V_m and time constant τ?

Substituting ② into ①, where $\frac{dv}{dt} = +\frac{V_m}{\tau} e^{-t/\tau}$

$$① \rightarrow \ m\frac{V_m}{\tau} e^{-t/\tau} = mg - \frac{V_m B^2 l^2}{R}\left(1 - e^{-t/\tau}\right)$$

$$\frac{V_m B^2 l^2}{R} = mg$$

$$V_m = \frac{Rmg}{B^2 l^2}$$ TERMINAL VELOCITY

At terminal velocity, $\frac{dv}{dt} = 0$ so $mg = \frac{V_m l^2 B^2}{R}$

At $t=0$, $\frac{dv}{dt} = g = \frac{V_m}{\tau} e^{-t/\tau}$

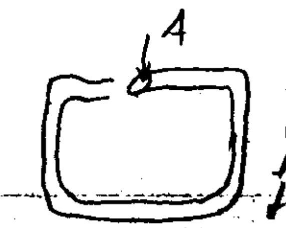
$$g = \frac{Rmg}{B^2 l^2 \tau} \frac{1}{e^0}$$

$$\tau = \frac{Rm}{B^2 l^2}$$ CHARACTERISTIC FALL TIME

Time to reach 90% of V_m? see next page.

If you cut a slit, no current can flow, so there's no $F_I = Il \times B$ and loop falls freely.

7.12 continued



terminal velocity $V_m = \frac{Rmg}{B^2 l^2}$. Let A = loop cross sectional area
L = loop's total length

Then $m = \rho A L$ where ρ = mass density $= 2.7\times10^3\ kg/m^3$
and $R = \frac{1}{\sigma}\frac{L}{A}$ where σ = electrical conductivity $= \frac{1}{(2.8\times10^{-8}\ \Omega\cdot m)}$
and $l = L/4$ is the length of one side of square loop

so $V_m = \left(\frac{L}{\sigma A}\right)\frac{(\rho A L)g}{B^2 (L/4)^2} = \frac{16\rho g}{\sigma B^2}$

$= \frac{16 (2.7\times10^3\ kg/m^3)\ 9.8\ m/s^2}{\left(\frac{1}{2.8\times10^{-8}\ \Omega m}\right)}$ [illegible] 1 Tesla = Volt·s/m² ... $= 1.19\ cm/s$

How long does it take to reach 90% of terminal velocity?

$v(t) = V_m(1 - e^{-t/\tau})$

$.9 V_m = V_m(1 - e^{-T/\tau})$ $\qquad T = (\ln 10)\ 1.21\ ms$

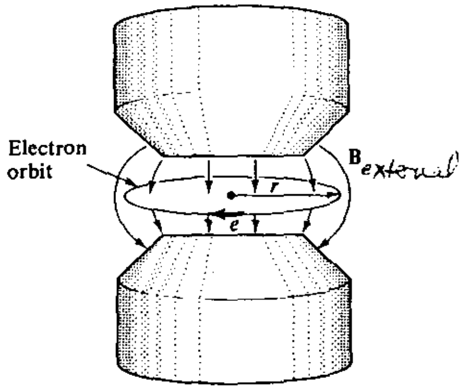
$1 - .9 = e^{-T/\tau}$ $\qquad = 2.79\ ms$

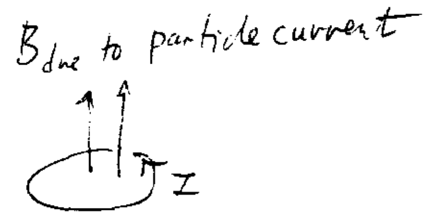
$\ln(\frac{1}{10}) = -\frac{T}{\tau}$

$\ln(\frac{1}{10}) = \frac{T}{\tau}$

$\tau = \frac{Rm}{B^2 l^2} = \frac{V_m}{g} = 1.21\ ms$

Problem 7.16 Electrons undergoing cyclotron motion can be speeded up by increasing the magnetic field; the accompanying electric field will impart tangential acceleration. This is the principle behind the **betatron**. One would like to keep the radius of the orbit constant during the process. Show that this can be achieved by designing a magnet such that the average field over the area of the orbit is twice the field at the circumference (Fig. 7.26). Assume the electrons start from rest in zero field, and that the apparatus is symmetric about the center of the orbit. (Assume also that the electron velocity remains well below the speed of light, so that nonrelativistic mechanics applies.)





The external magnetic field provides centripetal acceleration, so electrons orbit: $F = ma$

$$qvB = \frac{mv^2}{r} \rightarrow p = mv = qBr$$

If the external field strength increases toward the center, the induced Emf about one orbit will oppose the increasing linked flux by increasing opposing field due to particle current by increasing particle velocity.

$$\frac{d\Phi}{dt} = -\mathcal{E} = \int E \cdot dl = \frac{d}{dt}\int B \cdot da$$

$$E \cdot 2\pi r = \frac{d}{dt} B_{AV}\, \pi r^2 \rightarrow E = \frac{r}{2}\frac{d}{dt} B_{AV}$$

The Emf accelerates the electron: $F = \frac{dp}{dt}$

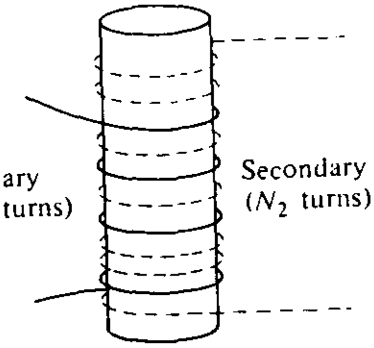
$$qE = \frac{d}{dt} qB(r)\, r$$

$$\underline{B_{AV} = 2B(r)} \leftarrow q\frac{r}{2}\frac{d}{dt}B_{AV} = \frac{d}{dt} q r B(r)$$

Problem 7.22 Two solenoidal coils are wrapped around a cylinder in such a way that the same amount of flux passes through every turn. (In practice this is achieved by inserting an iron core through the cylinder, which has the effect of concentrating the flux.) The "primary" coil has N_1 turns and the "secondary" N_2 (Fig. 7.36). If the current I in the primary is changing, show that the emf induced in the secondary is given by

$$\frac{\mathcal{E}_2}{\mathcal{E}_1} = \frac{N_2}{N_1} \tag{7.29}$$

where $\mathcal{E}_1$ is the (back) emf in the primary. (What we have here is a primitive **transformer**—a device for raising or lowering the emf of an alternating current source. By choosing the appropriate number of turns, any desired secondary emf can be obtained. If you think this violates conservation of energy, see Problem 7.58.)



Both coils link the same flux

$\Phi_1 = \Phi_2$ due to iron core

$$\frac{d\Phi_1}{dt} = \frac{d\Phi_2}{dt} = -\mathcal{E} \text{ per turn}$$

$$\frac{\mathcal{E}_1}{N_1} = \frac{\mathcal{E}_2}{N_2}$$

More turns → higher voltage (eg solenoid in car: steps 12 V up to ~ 20,000 V)

$$\mathcal{E}_2 = \mathcal{E}_1 \frac{N_2}{N_1}$$

If power in = power out, then $P_1 = P_2$

$$I_1\mathcal{E}_1 = I_2\mathcal{E}_2$$

More turns → lower current $I_2 = I_1 \dfrac{N_1}{N_2}$

Example 12

Find the vector potential of an infinite solenoid with N turns per unit length, radius R, and current I.

Solution: This time we cannot use (5.58), since the current itself extends to infinity. But here's a cute method that does the job. Notice that

$$\oint \mathbf{A} \cdot d\mathbf{l} = \int (\nabla \times \mathbf{A}) \cdot d\mathbf{a} = \int \mathbf{B} \cdot d\mathbf{a} = \Phi \tag{5.63}$$

where Φ is the flux of $\mathbf{B}$ through the loop in question. This is reminiscent of Ampère's law in integral form (5.49),

$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$$

In fact, it's the same equation, with $\mathbf{B} \rightarrow \mathbf{A}$ and $\mu_0 I_{enc} \rightarrow \Phi$. If symmetry permits, we can determine $\mathbf{A}$ from Φ in the same way we got $\mathbf{B}$ from I_{enc} in Section 5.3.3. The present problem (with a uniform longitudinal magnetic field $\mu_0 NI$ inside the solenoid and no field outside) is analogous to the Ampère's law problem of a fat wire carrying a uniformly distributed current. The vector potential is "circumferential" (it mimics the magnetic field of the wire); using a circular "amperian loop" at radius r *inside* the solenoid, we have

$$\oint \mathbf{A} \cdot d\mathbf{l} = A(2\pi r) = \int \mathbf{B} \cdot d\mathbf{a} = \mu_0 NI(\pi r^2)$$

so

$$\mathbf{A} = \frac{\mu_0 NI}{2} r\hat{\boldsymbol{\phi}}, \qquad \text{for } r < R \tag{5.64}$$

For an amperian loop *outside* the solenoid, the flux is

$$\int \mathbf{B} \cdot d\mathbf{a} = \mu_0 NI(\pi R^2)$$

since the field only extends out to R. Thus,

$$\mathbf{A} = \frac{\mu_0 NI}{2} \frac{R^2}{r} \hat{\boldsymbol{\phi}}, \qquad \text{for } r > R \tag{5.65}$$

If you have any doubts about this answer, *check* it: Does $\nabla \times \mathbf{A} = \mathbf{B}$? Does $\nabla \cdot \mathbf{A} = 0$? If so, we're done.

4M

Problem 7.25 Find the energy stored in a section of length l of a long solenoid (radius R, current I, N turns per unit length),

(a) Using (7.30) (you found L in Problem 7.20). (c) All space

(b) Using (7.31) (we worked out $\mathbf{A}$ in Example 12, Chapter 5). (d) tube outside solenoid

In class I showed that solenoid field $\bar{B} = \mu_0 N I \hat{k}$

and self-inductance $\frac{L}{l} = \frac{\Phi N}{I} = \frac{\mu_0 N^2 I A}{I} = \mu_0 N^2 A = \mu_0 N^2 \pi R^2$

So the energy stored in the solenoid (per unit length)

(7.30) $\boxed{W = \frac{1}{2}LI^2}$ $\frac{W}{l} = \frac{1}{2}\frac{L}{l}I^2 = \frac{\mu_0}{2}\pi R^2 N^2 I^2$

(b) The solenoid vector potential from Ex 12 Ch 5 is

$\bar{A}_{in} = \frac{\mu_0 N I}{2} r \hat{\varphi}$, so using $W = \frac{1}{2}\oint_C (\mathbf{A}\cdot\mathbf{I})\,dl$ (7.31)

$\underline{W} = \frac{1}{2}\int \bar{A}(R) I \underset{\#\text{ of turns}}{n} \cdot dl = \frac{1}{2}\frac{\mu_0 N I}{2} R I N l (2\pi R) = \underline{\frac{\mu_0}{2}\pi R^2 N^2 I^2 l}$

(c) if we agree to integrate over *all* space, then the surface integral goes to zero, and we are left with

$$\boxed{W = \frac{1}{2\mu_0}\int_{\text{all space}} B^2\,d\tau} \qquad (7.35)$$

$B_{in} = \mu_0 N I$

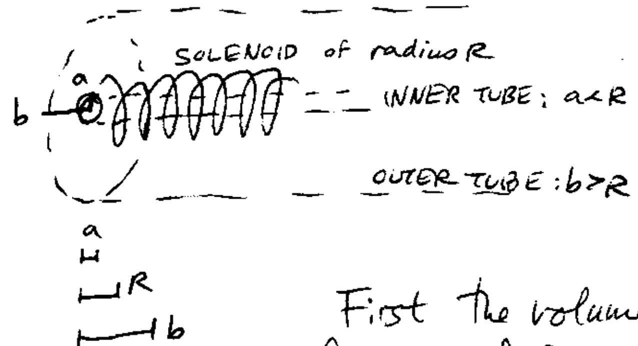
$B_{out} = 0$

$W = \frac{1}{2\mu_0} B_{in}^2 \cdot \text{volume} = \frac{1}{2\mu_0}(\mu_0 N I)^2 (\pi R^2 l) = \underline{\frac{\mu_0}{2}\pi R^2 N^2 I^2 l}$

(d)

$$W = \frac{1}{2\mu_0}\left[\int B^2\,d\tau - \int \nabla\cdot(\mathbf{A}\times\mathbf{B})\,d\tau\right]$$
$$= \frac{1}{2\mu_0}\left[\int_{\text{volume}} B^2\,d\tau - \oint_{\text{surface}} (\mathbf{A}\times\mathbf{B})\cdot d\mathbf{a}\right] \qquad (7.34)$$

Now, the integration (in 7.32) is to be taken over the *entire volume occupied by the current*. But any region *larger* than this will do just as well, for **J** is zero out there anyway. In the form (7.34), the larger the region we pick, the greater is the contribution from the B^2 integral and therefore the smaller is that of the surface integral (this makes sense: as the surface gets farther from the current, both **A** and **B** decrease). In

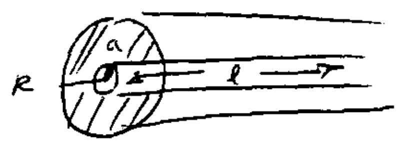


First the volume integral: $B_{out} = 0$

$$\int_{vol} B^2 d\tau = \int B_{in}^2\, d\tau_{in} + \int \cancel{B_{out}^2}\, d\tau_{out}$$

Volume of tube between $(a<r<R)$

$$\tau_{in} = l\,(\pi R^2 - \pi a^2)$$



$$\int_{vol} B^2 d\tau = (\mu_0 N I)^2\, l\,(\pi R^2 - \pi a^2)$$

Now the surface integral, over surfaces at $r=a$ and $r=b$:

$$\oint_{surface} (\bar{A}\times\bar{B})\cdot da = \int^{r=b} (A_{out}\times \cancel{B_{out}})^{0}\cdot da_{out} + \int^{r=a} (\bar{A}_{in}\times\bar{B}_{in})\cdot da_{in}$$

$$= \int_{\phi=0}^{2\pi} \left(\frac{\mu_0 N I}{2} a\hat{\phi}\right)\times(\mu_0 N I \hat{k})\cdot(l a\, d\phi\, \hat{r}) = \frac{\mu_0^2 N^2 I^2}{2}\, 2\pi l a^2$$

these cancel

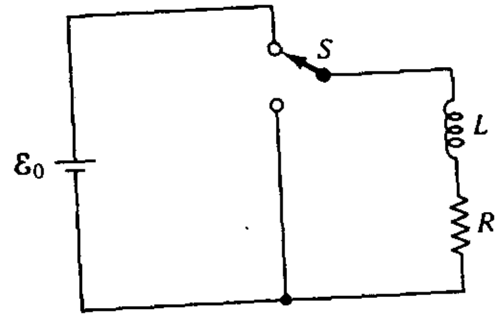
Finally, $W = \frac{1}{2\mu_0}\left[\int B^2 d\tau - \oint (A\times B)\cdot da\right] = \frac{1}{2\mu_0}\left[(\mu_0 N I)^2 l(\pi R^2 - \pi a^2) + (\mu_0 N I)^2 l \pi a^2\right]$

$$\frac{W}{l} = \frac{\mu_0}{2} N^2 I^2 \pi R^2$$

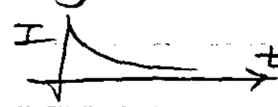
19

Problem 7.28 Suppose the circuit in Fig. 7.39 has been connected for a long time when suddenly, at time $t = 0$, switch S is thrown, bypassing the battery.
(a) What is the current at any subsequent time t?
(b) What is the total energy delivered to the resistor?
(c) Show that this is equal to the energy originally stored in the inductor.

In the original circuit, current flows freely & steadily, and inductor stores the maximum magnetic energy $W = \frac{1}{2} L I_{max}^2$



(a) After the switch is thrown, the inductor opposes change in its flux and sustains current as long as it can: $V_{loop} = L\frac{dI}{dt} - IR = 0$



$\frac{dI}{dt} = \frac{R}{L} I$ has solution $I = I_{max} e^{-t/\tau}$

where $\tau = L/R$

(b) Resistor dissipates energy W_R ohmically until $I \to 0$

$P = V_R(t) I(t) = \frac{dW_R}{dt} \to W_R = \int_0^\infty R I^2 dt = R I_{max}^2 \int e^{-2t/\tau} dt$

$\int e^{-2t/\tau} dt = -\frac{\tau}{2} e^{-t/\tau}\Big|_0^\infty = -\frac{\tau}{2}\left(\frac{1}{e^\infty} - \frac{1}{e^0}\right) = -\frac{\tau}{2}(0-1) = \frac{\tau}{2}$

Total energy dissipated by resistor $W_R = \frac{\tau}{2} R I_{max}^2$

(c) $W_L = \frac{1}{2} L I^2 = \frac{(R\tau)}{2} I^2 = W_R$ ✓

Problem 7.59 In a **perfect conductor**, the conductivity is infinite, so $E = 0$ (equation 7.3), and any net charge resides on the surface (just as it does for an *im*perfect conductor, in electro*statics*).

(a) Show that the magnetic field is constant (i.e., $\partial \mathbf{B}/\partial t = 0$), inside a perfect conductor.

FARADAY'S LAW: $\frac{\partial \bar{B}}{\partial t} = -\bar{\nabla}\times\bar{E}$ if $\bar{E}=0$ then $\nabla\times\bar{E}$

So $\bar{B}$ = constant inside a conductor.

A **superconductor** is a perfect conductor with the *additional* property that this constant **B** is always *zero*. (This "flux exclusion" is known as the **Meissner effect.**)

(b) Show that the current in a superconductor is confined to the surface.

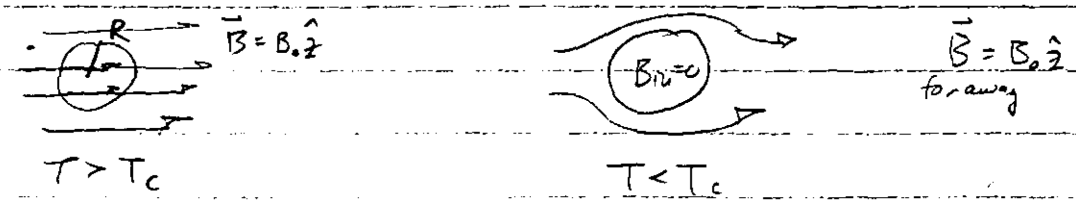
AMPERE'S LAW: $\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{enclosed}$.

If there were current inside the conductor, then you could draw an amperian loop INSIDE which enclosed nonzero I, and B_{in} would be nonzero.

Since $B_{in} = 0$, therefore $I_{in} = 0$

Superconductivity is lost above a certain critical temperature (T_c), which varies from one material to another.

(c) Suppose you had a sphere (radius R) above its critical temperature, and you held it in a uniform magnetic field $B_0\hat{z}$ while cooling it below T_c. Find the induced surface current density **K**, as a function of the polar angle θ.



Flux is expelled as the material cools below T_c.

To find $\bar{K}$, apply BC: $H E_{tan} \rightarrow \frac{B_{out\parallel}}{\mu_{out}} - \frac{B_{in\parallel}}{\mu_{in}} = K$

$BD_{perp} \rightarrow B_{\perp out} = B_{\perp in}$

Far away $(r\to\infty)$ $\bar{B} = B_0 \hat{k} = -\mu_0 \nabla \varphi^* = -\mu_0 \left[\frac{\partial \varphi^*}{\partial z}\hat{k}\right]$

so the dominant term in the φ^* series is

$$\varphi_1^* = -\frac{B_0}{\mu_0} z = -\frac{B_0}{\mu_0} r\cos\theta$$

Since B is finite at $r\to\infty$, $\varphi(r\to\infty)$ const blows up $(A_{l>1}=0)$

$$\varphi^*_{out} = \varphi_1^* + \sum_{l=0}^{\infty} \frac{C_l}{r^{l+1}} P_l = \frac{-B_0}{\mu_0} r\cos\theta + \sum \frac{C_l}{r^{l+1}} P_l$$

Substitute this into $\underline{\vec{B} = -\mu_0 \nabla \varphi^*}$ to find form of $\vec{B}$:

$$\bar{B}_{out} = -\mu_0 \bar{\nabla}\left[\frac{-B_0}{\mu_0} r\cos\theta + \sum \frac{C_l}{r^{l+1}} P_l\right]$$

$$B_{r_{out}} = -\mu_0 \frac{\partial}{\partial r}\left[\frac{-B_0}{\mu_0} r\cos\theta + \sum \frac{C_l}{r^{l+1}} P_l\right]$$

$$= B_0 \cos\theta + \mu_0 \sum \frac{(l+1) C_l P_l}{r^{l+2}}$$

Apply this to boundary at $r=a$:

$$B_r(a) = B_0 \cos\theta + \mu_0 \sum \frac{(l+1) C_l P_l}{a^{l+2}}$$

Since $B_\perp = B_\perp$ = continuous across boundary, the only term here is $P_1 = \cos\theta$ – all other l's vanish:

EMT HW #13 Ch 7 # 41(a), 59 Zita

Ch 8 # 9, 10, 11

8.1

Physics 335 - Electromagnetic Theory - Wed 7 Dec 94

Problem 7.41 Calculate the power (energy per unit time) transported down the cables of (a) Example 13 and Problem 7.29(b) assuming the two conductors are held at a potential difference V, and carry current I (down one and back up the other).

Example 13 (a)

A long coaxial cable carries current I (the current flows down the surface of the inner cylinder, radius a, and back along the outer cylinder, radius b) as shown in Fig. 7.38. Find the energy stored in a section of length l.

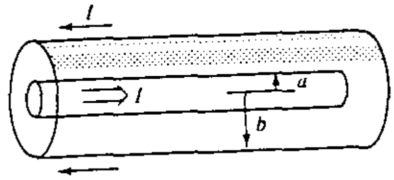


Figure 7.38

Solution: According to Ampère's law, the field between the cylinders is

$$\mathbf{B} = \frac{\mu_0 I}{2\pi r}\,\hat{\phi}$$

And the electric field between the two cylinders is

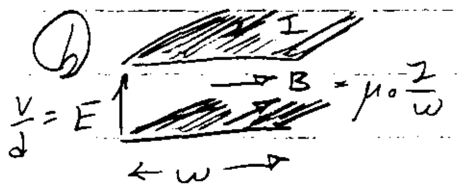
$$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 r}\,\hat{r} \quad \text{and} \quad V = -\int_a^b \bar{E}\cdot dr = \frac{\lambda}{2\pi\epsilon_0}\ln\frac{b}{a} = rE\ln\frac{b}{a}$$

$$\text{Intensity } S = \frac{\text{Power}}{\text{area}} = \frac{1}{\mu_0}|E\times B|$$

$$\text{Power} = \int S\cdot da = \frac{1}{\mu_0}\int EB\, r\,dr\,d\theta = \frac{2\pi}{\mu_0}\int E\frac{\mu_0 I}{2\pi r}r\,dr$$

$$= I\int_a^b E\,dr = IV \checkmark$$

(b)



Long current-carrying ribbons of width w separated by d:

$$\text{Power} = \int S\,da = \frac{1}{\mu_0}EB(dw) = \frac{1}{\mu_0}\left(\frac{V}{d}\right)\left(\mu_0\frac{I}{w}\right)wd = IV \checkmark$$

Problem 8.10 The intensity of sunlight hitting the earth is about 1300 W/m^2. Find the amplitude of the electric and magnetic fields. How far from a stationary electron would you have to be to get a comparable electric field? (Actually, the sun's radiation is neither monochromatic nor linearly polarized, so don't take these numbers too seriously.) If the sunlight falls on a perfect absorber (which soaks up all the incident momentum), what pressure does it exert? How about a perfect reflector? What fraction of atmospheric pressure does this amount to?

Intensity $\langle \bar{S} \rangle = \frac{\text{Power}}{\text{area}} = \frac{1}{\mu_0} \langle \vec{E} \times \vec{B} \rangle = \frac{1}{2} c\epsilon_0 E_0^2$ for monochromatic plane waves

Let's use this, even though sunlight is not monochromatic (it consists of many wavelengths, not just one)

At earth, $E_{\text{due to sunlight}} = \sqrt{\frac{2\langle S \rangle}{c\epsilon_0}} = \sqrt{\frac{2 \cdot 1300\ W/m^2}{3 \cdot 10^8 \frac{m}{s} \cdot 8.85 \cdot 10^{-12} \frac{C^2}{Nm^2}}}$

$= \sqrt{\frac{1.3 \times 10^{3-8+12}}{1.5 \times .885 \times 10}} \simeq \sqrt{1 \times 10^{7-1}} \simeq 10^3 \frac{V}{m}$

and $B = \frac{10^3 \frac{V}{m}}{10^6 \frac{m}{s}} = 10^{-3} \left(\frac{V \cdot s}{m^2} = \text{Tesla} \right)$

(b) Field due to stationary electron $E = \frac{kq}{r^2}$ has the same strength at a distance

$R = \sqrt{\frac{kq}{E}} = \sqrt{\frac{9 \times 10^9 \times 1.6 \times 10^{-19}}{10^3 V/m}} \simeq \sqrt{10^{1+9-19-3}} = \sqrt{10^{-12}}$

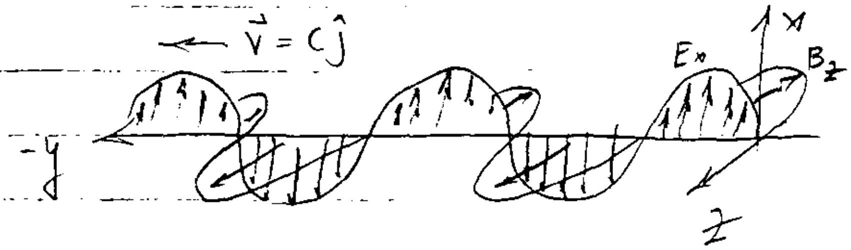
$R \simeq 10^{-6}$ m

checking units: (a) $\sqrt{\frac{m}{s} \frac{V \cdot s/m}{A} \frac{VA}{m^2}} = \sqrt{\frac{V^2}{m^2}} = \frac{V}{m}$ ✓

(b) $\sqrt{\frac{Nm^2}{C^2} \frac{C}{V/m}} = \sqrt{\frac{J \cdot m^2}{C \cdot V}} = \sqrt{m^2} = m$ ✓

9.9

Problem 8.9 Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is
(a) traveling in the negative y-direction and polarized in the x-direction;
(b) traveling in the direction from the origin to the point (1, 1, 1), with polarization parallel to the xy plane.
In each case, sketch the wave, and give the explicit Cartesian components of κ and $\hat{n}$.



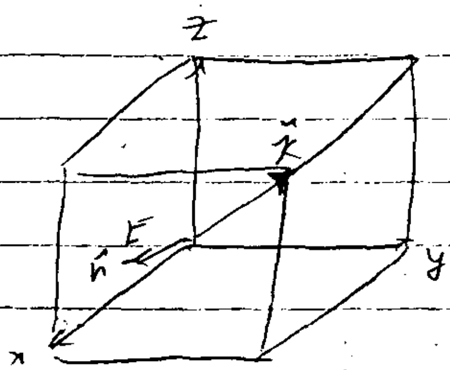
$$\vec{E} = E_0 \sin(ky + \omega t)\,\hat{i}$$
$$\vec{B} = -\frac{E_0}{c}\sin(ky + \omega t)\,\hat{k}$$

$$\vec{E} \times \vec{B} = \vec{v} \rightarrow B = \frac{E_0}{c}$$

The wave vector $\vec{k} = -\frac{\omega}{c}\,\hat{j}$ since the wave propagates in the $-y$ direction

The polarization vector is the direction of the $\vec{E}$ field : $\hat{n} = \hat{i}$

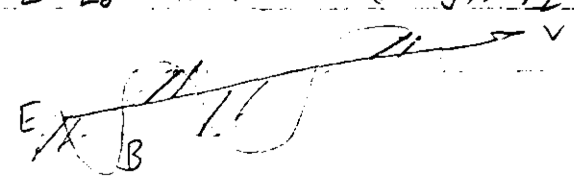
(b)



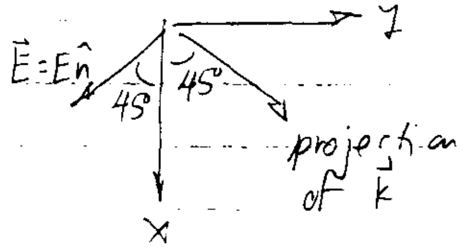
$$\vec{k} = \frac{\omega/c}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$$

$$\hat{n} = \frac{1}{\sqrt{2}}(\hat{i} - \hat{j})$$ by inspection: See top VIEW

$$\vec{E} = E_0 \sin(\vec{k}\cdot\vec{r} - \omega t)\,(\hat{i} - \hat{j})/\sqrt{2}$$



TOP VIEW



$$\vec{B} = \frac{\hat{k} \times \vec{E}}{c} = \frac{E_0}{\sqrt{6}\,c}(\hat{i} + \hat{j} + \hat{k}) \times (\hat{i} - \hat{j})\sin(\vec{k}\cdot\vec{r} - \omega t)$$

$$\vec{B} = \frac{E_0}{\sqrt{6}\,c}\sin(\vec{k}\cdot\vec{r} - \omega t)\,(\hat{i} + \hat{j} - 2\hat{k})$$

Problem 8.11 In the complex notation there is a cute trick for finding the average of a product. Suppose $f(\mathbf{r}, t) = a\cos(\boldsymbol{\kappa}\cdot\mathbf{r} - \omega t + \delta_a)$ and $g(\mathbf{r}, t) = b\cos(\boldsymbol{\kappa}\cdot\mathbf{r} - \omega t + \delta_b)$. Show that $\langle fg\rangle = (1/2)\,\mathrm{Re}(\tilde{f}\tilde{g}^*)$. Thus for example

$$\langle U\rangle = \frac{1}{4}\,\mathrm{Re}\left(\epsilon_0\tilde{\mathbf{E}}\cdot\tilde{\mathbf{E}}^* + \frac{1}{\mu_0}\tilde{\mathbf{B}}\cdot\tilde{\mathbf{B}}^*\right)$$

where $\tilde{E} = Ee^{i\delta_E}$, $\tilde{B} = Be^{i\delta_B}$

$$\langle \mathbf{S}\rangle = \frac{1}{2\mu_0}\,\mathrm{Re}\,(\tilde{\mathbf{E}}\times\tilde{\mathbf{B}}^*)$$

$\langle f\rangle = \dfrac{\int_0^T f\,dt}{T}$ so

① $\langle fg\rangle = \int_0^T ab\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_a)\cos(\vec{k}\cdot\vec{r} - \omega t + \delta_b)\,dt \,/\, T$

In complex notation $f = \mathrm{Re}\left(ae^{i(\vec{k}\cdot\vec{r} - \omega t + \delta_a)}\right) = \mathrm{Re}(\tilde{f})$

where $\tilde{f} = \tilde{a}e^{i(\vec{k}\cdot\vec{r} - \omega t)}$ and the phase factor δ_a is absorbed in the complex amplitude $\tilde{a} = ae^{i\delta_a}$

Write the result we seek in real terms:

$$\tfrac{1}{2}\mathrm{Re}(\tilde{f}\tilde{g}^*) = \tfrac{1}{2}\mathrm{Re}\left(\tilde{a}e^{i(\vec{k}\cdot\vec{r} - \omega t)}\tilde{b}^* e^{-i(\vec{k}\cdot\vec{r} - \omega t)}\right)$$
$$= \tfrac{1}{2}\mathrm{Re}(\tilde{a}\tilde{b}^* e^0)$$
$$= \tfrac{1}{2}\mathrm{Re}(ae^{i\delta_a}be^{-i\delta_b}) = \tfrac{1}{2}ab\,\mathrm{Re}\left(e^{i(\delta_a - \delta_b)}\right)$$

② $\langle fg\rangle = \tfrac{1}{2}ab\cos(\delta_a - \delta_b)$ AVERAGE DEPENDS ON PHASE DIFFERENCE

Compare to $\langle fg\rangle$, rewriting integrand using $\cos(A+B) = \cos A\cos B - \sin A\sin B$ TRIG IDENTITY

$\cos(\bar{k}\cdot\bar{r} - \omega t + \delta_a) = \cos(\bar{k}\cdot\bar{r} - \omega t)\cos\delta_a - \sin(\bar{k}\cdot\bar{r} - \omega t)\sin\delta_a$

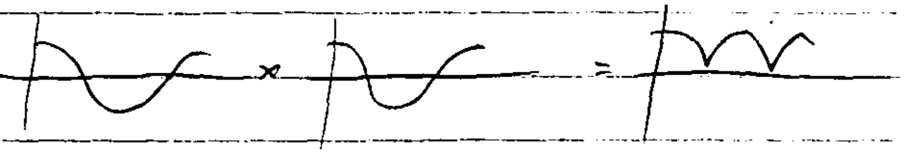
$\cos(\bar{k}\cdot\bar{r} - \omega t + \delta_b) = \cos(\bar{k}\cdot\bar{r} - \omega t)\cos\delta_b - \sin(\bar{k}\cdot\bar{r} - \omega t)\sin\delta_B$

$= \cos x\cos\delta_b - \sin x\sin\delta_B$

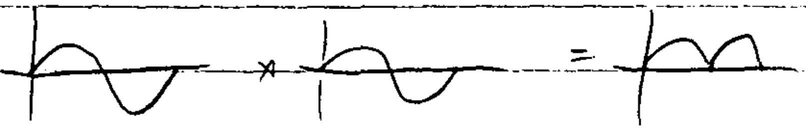
constant; no t-dependence; comes outside integral

$\cos(x+\delta_a)\cos(x+\delta_b) = [\cos x \cos\delta_a - \sin x \sin\delta_a][\cos x \cos\delta_b - \sin x \sin\delta_b]$

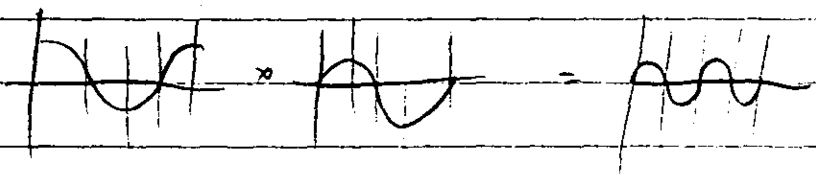
Notice which terms survive upon integration over a period T:



$\int_0^T \cos^2 x\, dt = \frac{T}{2}$



$\int_0^T \sin^2 x\, dt = \frac{T}{2}$



$\int_0^T \cos x \sin x\, dt = 0$

So $\int_0^T [\cos^2 x \cos\delta_a \cos\delta_b + \sin^2 x \sin\delta_a \sin\delta_b + \underbrace{\text{cross-terms}}_{\text{which vanish upon integration}}]\, dt$

$= \frac{T}{2}(\cos\delta_a \cos\delta_b + \sin\delta_a \sin\delta_b) = \frac{T}{2}\cos(\delta_a - \delta_b)$ looks familiar!

Then $\langle fg \rangle = ab\, \frac{T}{2}\cos(\delta_a - \delta_b)/T$ from ①

$= \frac{ab}{2}\cos(\delta_a - \delta_b)$

$= \frac{1}{2}\operatorname{Re}(\tilde{f}\tilde{g}^*)$ from ② ✓

(Complex conjugate → $\ominus i\delta_b$ → phase difference)

Physics 335 EMT HW 12 Nov 93 Ch 10 #44 Ch 5 #1-5 ZITA

#8

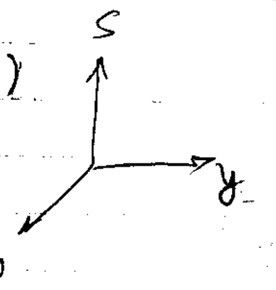
Problem 10.44

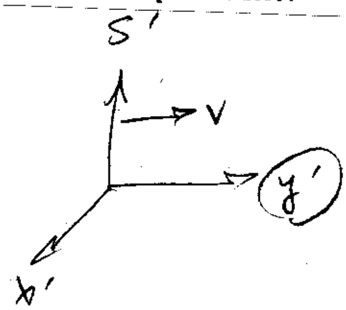
(a) Show that $(\mathbf{E}\cdot\mathbf{B})$ is relativistically invariant.
(b) Show that $(E^2 - c^2B^2)$ is relativistically invariant.
(c) Suppose the *magnetic* field at some point is *zero* in one system. Is it possible to find another system in which the *electric* field at that point is zero?

$x = \gamma(x' + vt')$

$y = y'$

$t = \gamma\left(t' + \frac{vx'}{c^2}\right)$



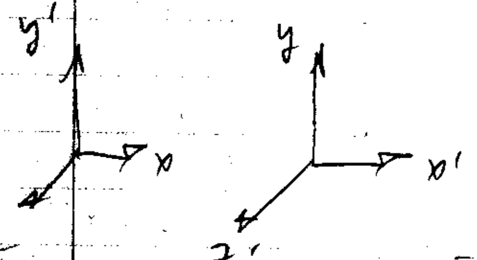


$x' = \gamma(x - vt)$

$y' = y$

$t' = \gamma\left(t - \frac{v}{c^2}x\right)$

$F_\perp' = \frac{F_\perp}{\gamma}$ $\quad F_\parallel' = F_\parallel$



$$E_x' = E_x, \quad E_y' = \gamma(E_y - vB_z), \quad E_z' = \gamma(E_z + vB_y)$$
$$B_x' = B_x, \quad B_y' = \gamma\left(B_y + \frac{v}{c^2}E_z\right), \quad B_z' = \gamma\left(B_z - \frac{v}{c^2}E_y\right) \qquad (10.119)$$

$\bar{E}\cdot\bar{B} = E_x B_x + E_y B_y + E_z B_z$

$\bar{E}'\cdot\bar{B}' = E_x B_x + \gamma^2(E_y - vB_z)\left(B_y + \frac{vE_z}{c^2}\right) + \gamma^2(E_z + vB_y)\left(B_z - \frac{vE_y}{c^2}\right)$

$= E_x B_x + \gamma^2\Big[\left(E_yB_y + \frac{v}{c^2}E_yE_z - vB_zB_y - \frac{v^2}{c^2}B_zE_z\right)$
$\left(-\frac{v^2}{c^2}E_yB_y - \frac{v}{c^2}E_zE_y + vB_yB_z + E_zB_z\right)\Big]$

$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$\gamma^2 E_yB_y\left(1 - \frac{v^2}{c^2}\right) = E_yB_y \left(1-\frac{v^2}{c^2}\right)\Big/\left(1-\frac{v^2}{c^2}\right) = E_yB_y.$

Likewise, $\gamma^2 E_zB_z\left(1-\frac{v^2}{c^2}\right) = E_zB_z$

So $\bar{E}'\cdot\bar{B}' = E_xB_x + \gamma^2\left[E_yB_y\left(1-\frac{v^2}{c^2}\right) + 0 + E_zB_z\left(1-\frac{v^2}{c^2}\right)\right]$
$= E_xB_x + E_yB_y + E_zB_z$
$= \bar{E}\cdot\bar{B}$ ✓ relativistically invariant

$\boxed{10.44(c)}$ If $B_x = B_y = B_z = 0$ then

$E_x' = E_x \quad E_y' = \gamma(E_y - 0) \quad E_z' = \gamma(E_z + 0)$

$\vec{E}' = 0$ (in moving frame) ONLY IF $\vec{E} = 0$ (in original frame)

NO NONTRIVIAL SOLUTIONS: If $\bar{B} = 0$ in one system, then ($\bar{E} \neq 0$ in all systems unless $\bar{E} = 0$ in all systems)