Problem 8.11[10] Picture the electron as a uniformly charged spherical shell, with charge e and radius R, spinning at angular velocity ω.

(a) Calculate the total energy contained in the electromagnetic fields. $W = W_{B_{in}} + W_{B_{out}} + W_E$

(b) Calculate the total angular momentum contained in the fields.

(c) According to the Einstein formula ($E = mc^2$), the energy in the fields should contribute to the mass of the electron. Lorentz and others speculated that the *entire* mass of the electron might be accounted for in this way: $U_{em} = m_e c^2$. Suppose, moreover, that the electron's spin angular momentum is entirely attributable to the electromagnetic fields: $L_{em} = \hbar/2$. On these two assumptions, determine the radius and angular velocity of the electron. What is their product, ωR? Does this classical model make sense?

Example 5.11 found $\mathbf{B} = \frac{2}{3}\mu_0 \sigma R \omega$ for this problem (5.68) inside $r<R$ 237

$\sigma = \frac{e}{4\pi R^2}$

Since $q_{inside} = 0$, $\mathbf{E}(r<R) = 0$

Problem 5.36 found $\mathbf{B}(r>R) = \frac{\mu_0}{4\pi}\frac{m}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta})$

where $m = \frac{4}{3}\pi \sigma \omega R^4$.

Gauss' Law tells us that $\mathbf{E}(r>R) = \frac{1}{4\pi\epsilon_0}\frac{e}{r^2}\hat{r}$

Example 2.8 (94) found the energy stored in the electric field $W_E = \frac{e^2}{8\pi\epsilon_0 R}$

Energy density of the magnetic field INSIDE $(r<R)$

$$u_{B_{in}} = \frac{B_{in}^2}{2\mu_0} = \frac{\frac{2^2}{3^2}\mu_0^2\sigma^2R^2\omega^2}{2\mu_0} = \frac{2}{9}\mu_0 R^2\omega^2\left(\frac{e}{4\pi R^2}\right)^2$$

$$u_{B_{in}} = \frac{2}{9}\frac{\mu_0\omega^2e^2}{16\pi^2R^2}, \quad \text{So } W_{B_{in}} = u_{B_{in}}\frac{4}{3}\pi R^3$$

$$W_{B_{in}} = \frac{\mu_0\omega^2e^2}{9\cdot 8\pi^2R^2}\frac{4\pi R^3}{3} = \frac{\mu_0\omega^2e^2R}{27\cdot 2\pi}$$

$$u_{B_{out}} = \frac{B_{out}^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0}{4\pi}\frac{4\pi\sigma\omega R^4}{3}\frac{}{r^3}\right)^2\left(4\cos^2\theta + \sin^2\theta\right)$$

$$= \frac{\mu_0}{2}\,\frac{\omega^2 R^8}{9 r^6}\left(3\cos^2\theta + 1\right)\left(\sigma = \frac{e}{4\pi R^2}\right)^2$$

$$= \frac{\mu_0 \omega^2 R^8 e^2}{2\cdot 9 r^6 16\pi^2 R^4}\left(3\cos^2\theta + 1\right) = \frac{\mu_0 \omega^2 R^4 e^2 (3\cos^2\theta + 1)}{32\cdot 9 r^6 \pi^2}$$ ✓

$$W_{B\,out} = \int u_{B_{out}}\, d\mathrm{vol} = \int u_{B_{out}}\, r^2 dr \sin\theta\, d\theta\, d\phi$$

$$= \frac{\mu_0 \omega^2 R^4 e^2}{32\cdot 9\pi^2}\int_0^\infty \frac{r^2 dr}{r^6}\int_0^\pi (3\cos^2\theta + 1)\sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$\int \frac{r^2 dr}{r^6} = \int r^{-4} dr = -\frac{1}{3} r^{-3}\Big|_R^\infty = -\frac{1}{3}\left(\frac{1}{r^\infty} - \frac{1}{R^3}\right) = \frac{1}{3R^3}$$

$$\int_0^\pi (3\cos^2\theta + 1)\sin\theta\, d\theta = \int_0^\pi 3\cos^2\theta \sin\theta\, d\theta + \int_0^\pi \sin\theta\, d\theta$$

$$= 3\left(-\frac{\cos^3\theta}{3}\right) - \cos\theta\Big|_0^\pi = -\cos^3\theta - \cos\theta\Big|_0^\pi = -\left[(-1-1) + (-1-1)\right] =$$

DWIGHT $\frac{450.12}{107}$

$$W_{B\,out} = \frac{2\pi \mu_0 \omega^2 R^4 e^2}{32\cdot 9\pi^2}\,\frac{1}{3R^3}\,4 = \frac{\mu_0 \omega^2\, e^2 R}{4\cdot 27\pi}$$ //

$$W_B = W_{B_{out}} + W_{B_{in}} = \frac{\mu_0 \omega^2 e^2 R}{27\pi}\left[\frac{1}{4} + \frac{1}{2} = \frac{3}{4}\right]$$

8.11.2 $$W_{tot} = W_E + W_B = \frac{e^2}{8\pi\epsilon_0 R} + \frac{\mu_0 \omega^2 e^2 R}{9\pi}\left[\frac{1}{4}\right]$$ /

(b) Calculate the total angular momentum in the fields

$\vec{l} = \vec{r} \times \vec{p}$ where $\vec{p} = \epsilon_0 (\vec{E} \times \vec{B}) = \frac{e}{4\pi\epsilon_0 r^2} \frac{\mu_0 m}{4\pi r^3} \sin\theta \, (\hat{r} \times \hat{\theta})$

$(r > R)$ $= \frac{e \mu_0 m \sin\theta}{\epsilon_0 r^5 4^2 \pi^2} \vec{\phi}$

$p(r < R) = 0$ because $\vec{E}(r < 0) = 0$

$\hat{l} =$ $(\hat{r} \times \hat{\phi})$ and $(\hat{r} \times \vec{\phi}) = -\hat{\theta}$

Only the z component will survive integration so, since $(\vec{\theta})_z = -\sin\theta$,

$\vec{L} = \int l \, r^2 \sin\theta \, dr \, d\theta \, d\phi$ where $\int_0^{2\pi} d\phi = 2\pi$, $\int_0^{\pi} \sin^3\theta \, d\theta = \frac{4}{3}$,

$\int_R^{\infty} \frac{1}{r^2} dr = \left(\frac{-1}{r}\right)_R^{\infty} = \frac{-1}{\infty} - \left(\frac{-1}{R}\right) =$

$\vec{L} =$

$\vec{L} = \frac{\mu_0 e^2 \omega R}{18\pi} \hat{z}$

(c) If $L = \frac{\hbar}{2} = \frac{\mu_0 e^2 \omega R}{18\pi}$ then $\omega R = \frac{9\pi\hbar}{\mu_0 e^2} = \frac{9\pi(10^{-34} J.s)}{(4\pi \times 10^{-7} \frac{Tm}{A})(1.6 \times 10^{-19} c)}$

$\omega R = 9 \times 10^{10} \frac{m}{s}$

8.11.3

If $W_{EB} = mc^2 = \frac{e^2}{8\pi\epsilon_0 R} + \frac{\mu_0 \omega^2 e^2 R}{4 \cdot 9\pi} = \frac{e^2}{8\pi\epsilon_0 R}\left[1 + \frac{2}{9}\omega^2 R^2 \mu_0 \epsilon_0\right]$

then $mc^2 = \frac{e^2}{8\pi\epsilon_0 R}\left[1 + \frac{2}{9}\frac{\omega^2 R^2}{c^2}\right]$

$$\left(\frac{\omega R}{c}\right)^2 = \left(\frac{9 \times 10^{10}\,\frac{m}{s}}{3 \times 10^{8}\,\frac{m}{s}}\right)^2 = (3 \times 10^2)^2 = 9 \times 10^4$$

$$[\;] = 1 + \frac{2}{9}\frac{\omega^2 R^2}{c^2} = 1 + 2 \times 10^4 \simeq 2 \times 10^4$$

$$R = \frac{e^2\,[2 \times 10^4]}{8\pi\epsilon_0 mc^2} = \frac{(1.6 \times 10^{-19}\,C)^2 \times 2 \times 10^4}{8\pi \times 8.85 \times 10^{-12}\,\frac{C^2}{N\,m^2} \times 9.11 \times 10^{-31}\,kg\left(3 \times 10^8\,\frac{m}{s}\right)^2}$$

$R \simeq 3 \times 10^{-11}\,m$

$\omega = \frac{\omega R}{R} \simeq \frac{9 \times 10^{10}\,\frac{m}{s}}{3 \times 10^{-11}\,m} \simeq 3 \times 10^{21}\,rad/s$ yow!

Since $\omega R \sim 300c$, we can not simply explain the mass of the electron in terms of its electromagnetic energy.

Prob 9.3. Determine A_3 & δ_3 in terms of $A_1, A_2, \delta_1,$ and δ_2.

p.369 Ex 9.1 Combine $f_3 = f_1 + f_2 = Re(\tilde{f}_1) + Re(\tilde{f}_2) = Re(\tilde{f}_1 + \tilde{f}_2) = Re(\tilde{f}_3)$

$$\tilde{A}_3 e^{i(kz-\omega t)} = \tilde{f}_3 = \tilde{f}_1 + \tilde{f}_2 = \tilde{A}_1 e^{i(kz-\omega t)} + \tilde{A}_2 e^{i(kz-\omega t)}$$

$$\left.\begin{aligned} \tilde{A}_3 &= \tilde{A}_1 + \tilde{A}_2 \\ A_3 e^{i\delta_3} &= A_1 e^{i\delta_1} + A_2 e^{i\delta_2} \end{aligned}\right\} (9.19)$$

$f_3 = A_3 \cos(kz - \omega t + \delta_3)$ - find A_3 and δ_3 from (9.19)

$$f_3 = f_1 + f_2 = A_1 \cos(kz - \omega t + \delta_1) + A_2 \cos(kz - \omega t + \delta_2)$$
$$= A_3 \cos(kz - \omega t + \delta_3)$$

B

Try squaring: $(A_3)^2 = (A_3 e^{-i\delta_3})(A_3 e^{i\delta_3})$ = GETS RID OF δ_3
$$= (A_1 e^{-i\delta_1} + A_2 e^{-i\delta_2})(A_1 e^{i\delta_1} + A_2 e^{i\delta_2})$$

Use trig identities and Euler's formula to show that
$e^{i(\delta_1 - \delta_2)} + e^{i(\delta_2 - \delta_1)} = 2\cos(\delta_1 - \delta_2)$ ... find A_3

To find δ_3, write $A_3 e^{i\delta_3} = A_3(\cos\delta_3 + i\sin\delta_3)$

$A_3 e^{i\delta_3} = A_1(\cos\delta_1 + i\sin\delta_1) + A_2(\cos\delta_2 + i\sin\delta_2) = (A_1\cos\delta_1 + A_2\cos\delta_2) + i(\ldots$

$$\tan\delta_3 = \frac{A_3 \sin\delta_3}{A_3\cos\delta_3} = \frac{Im}{Re} =$$