3. **(a)** The equilibrium solutions correspond to the values of P for which $dP/dt = 0$ for all t. For this equation, $dP/dt = 0$ for all t if $P = 0$ or $P = 230$.

(b) The population is increasing if $dP/dt > 0$. That is, $P(1 - P/230) > 0$. Hence, $0 < P < 230$.

(c) The population is decreasing if $dP/dt < 0$. That is, $P(1 - P/230) < 0$. Hence, $P > 230$ or $P < 0$. Since this is a population model, $P < 0$ might be considered "nonphysical."

4. **(a)** The equilibrium solutions correspond to the values of P for which $dP/dt = 0$ for all t. For this equation, $dP/dt = 0$ for all t if $P = 0$, $P = 50$, or $P = 200$.

(b) The population is increasing if $dP/dt > 0$. That is, $P < 0$ or $50 < P < 200$. Note, $P < 0$ might be considered "nonphysical" for a population model.

(c) The population is decreasing if $dP/dt < 0$. That is, $0 < P < 50$ or $P > 200$.

10. **(a)** The rate of change of the amount of radioactive material is dr/dt. This rate is proportional to the amount r of material present at time t. With $-\lambda$ as the proportionality constant, we obtain the differential equation

$$\frac{dr}{dt} = -\lambda r.$$

Note that the minus sign (along with the assumption that λ is positive) means that the material decays.

(b) The only additional assumption is the initial condition $r(0) = r_0$. Consequently, the corresponding initial-value problem is

$$\frac{dr}{dt} = -\lambda r, \quad r(0) = r_0.$$

2. We note that $dy/dt = 2e^{2t}$ for $y(t) = e^{2t}$. If $y(t) = e^{2t}$ is a solution to the differential equation, then we must have

$$\begin{aligned} 2e^{2t} &= 2y(t) - t + g(y(t)) \\ &= 2e^{2t} - t + g(e^{2t}). \end{aligned}$$

Hence, we need

$$g(e^{2t}) = t.$$

This equation is satisfied if we let $g(y) = (\ln y)/2$. In other words, $y(t) = e^{2t}$ is a solution of the differential equation

$$\frac{dy}{dt} = 2y - t + \frac{\ln y}{2}.$$

5. The constant function $y(t) = 0$ is an equilibrium solution.

For $y \neq 0$ we separate the variables and integrate

$$\begin{aligned} \int \frac{dy}{y} &= \int t\,dt \\ \ln|y| &= \frac{t^2}{2} + c \\ |y| &= c_1 e^{t^2/2} \end{aligned}$$

where $c_1 = e^c$ is an arbitrary positive constant.

If $y > 0$, then $|y| = y$ and we can just drop the absolute value signs in this calculation. If $y < 0$, then $|y| = -y$, so $-y = c_1 e^{t^2/2}$. Hence, $y = -c_1 e^{t^2/2}$. Therefore,

$$y = ke^{t^2/2}$$

where $k = \pm c_1$. Moreover, if $k = 0$, we get the equilibrium solution. Thus, $y = ke^{t^2/2}$ yields all solutions to the differential equation if we let k be any real number. (Strickly speaking we need a theorem from Section 1.5 to justify the assertion that this formula provides all solutions.)

19. The function $y(t) = 0$ for all t is an equilibrium solution.

Suppose $y \neq 0$ and separate variables. We get

$$\int y + \frac{1}{y}\,dy = \int e^t\,dt$$

$$\frac{y^2}{2} + \ln|y| = e^t + c,$$

where c is any real constant. We cannot solve this equation for y, so we leave the expression for y in this implicit form. Note that the equilibrium solution $y = 0$ cannot be obtained from this implicit equation.

24. First we find the general solution by writing the differential equation as

$$\frac{dy}{dt} = (t+2)y^2,$$

separating variables, and integrating. We have

$$\begin{aligned} \int \frac{1}{y^2}\,dy &= \int (t+2)\,dt \\ -\frac{1}{y} &= \frac{t^2}{2} + 2t + c \\ &= \frac{t^2 + 4t + c_1}{2}, \end{aligned}$$

where $c_1 = 2c$. Inverting and multiplying by -1 produces

$$y(t) = \frac{-2}{t^2 + 4t + c_1}.$$

Setting

$$1 = y(0) = \frac{-2}{c_1}$$

and solving for c_1, we obtain $c_1 = -2$. So

$$y(t) = \frac{-2}{t^2 + 4t - 2}.$$

35. **(a)** If we let k denote the proportionality constant in Newton's law of cooling, the differential equation satisfied by the temperature T of the chocolate is

$$\frac{dT}{dt} = k(T - 70).$$

We also know that $T(0) = 170$ and that $dT/dt = -20$ at $t = 0$. Therefore, we obtain k by evaluating the differential equation at $t = 0$. We have

$$-20 = k(170 - 70),$$

so $k = -0.2$. The initial-value problem is

$$\frac{dT}{dt} = -0.2(T - 70), \quad T(0) = 170.$$

(b) We can solve the initial-value problem in part (a) by separating variables. We have

$$\begin{aligned} \int \frac{dT}{T - 70} &= \int -0.2\,dt \\ \ln|T - 70| &= -0.2t + k \\ |T - 70| &= ce^{-0.2t}. \end{aligned}$$

Since the temperature of the chocolate cannot become lower than the temperature of the room, we can ignore the absolute value and conclude

$$T(t) = 70 + ce^{-0.2t}.$$

Now we use the initial condition $T(0) = 170$ to find the constant c because

$$170 = T(0) = 70 + ce^{-0.2(0)},$$

which implies that $c = 100$. The solution is

$$T = 70 + 100e^{-0.2t}.$$

In order to find t so that the temperature is 110° F, we solve

$$110 = 70 + 100e^{-0.2t}$$

for t obtaining

$$\begin{aligned} \tfrac{2}{5} &= e^{-0.2t} \\ \ln\tfrac{2}{5} &= -0.2t \end{aligned}$$

so that

$$t = \frac{\ln(2/5)}{-0.2} \approx 4.6.$$