15.5 ⋆ With $\beta = 0.95$, the γ factor for both the outward and return trips is $\gamma = 1/\sqrt{1-\beta^2} = 3.20$. The times for the two halves of the journey satisfy

$$\Delta t_B^{\text{out}} = \gamma \Delta t_A^{\text{out}} \quad \text{and} \quad \Delta t_B^{\text{back}} = \gamma \Delta t_A^{\text{back}},$$

so, by addition, the times for the whole journey satisfy the same relation $\Delta t_B = \gamma \Delta t_A$. Therefore $\Delta t_A = \Delta t_B/\gamma = (80 \text{ yr})/3.20 = 25$ yr, which is the amount by which twin A has aged.

15.6 ⋆ Clearly $\gamma = 1/\sqrt{1-\beta^2} = 3$, so $\beta = \sqrt{1 - 1/\gamma^2} = \sqrt{8/9} = 0.94$, and $v = 0.94c = 2.8 \times 10^8$ m/s.

15.8 ⋆⋆ **(a)** With $\beta = 4/5$, $\gamma = 5/3$. The half-life measured in the lab is $t_{1/2}(\text{lab}) = \gamma t_{1/2}(\text{proper}) = (5/3) \times (1.8 \times 10^{-8} \text{ s}) = 3.0 \times 10^{-8}$ s.

(b) The time of flight (measured in the lab) is $T(\text{lab}) = d/v$, where $d = 36$ m is the length of the pipe. Thus the number of half-lives elapsed is

$$n = \frac{T(\text{lab})}{t_{1/2}(\text{lab})} = \frac{d/v}{\gamma t_{1/2}(\text{proper})} = 5.00. \qquad \text{(v)}$$

Therefore, the number of pions that survive the journey is $N = N_o/2^n = 32,000/2^5 = 1000$.

(c) To find the classical answer, we must delete the factor of γ in the expression (v) for n, to give $n(\text{clas}) = \gamma n(\text{rel}) = 8.33$ half-lives, and $N(\text{clas}) = 32,000/2^{8.33} \approx 100$.

15.11 ⋆ The stick's proper length is $l_o = 100$ cm, whereas I measure it to be $l = 80$ cm. Since $l = l_o/\gamma$, we see that $\gamma = 5/4$. Because $\gamma = 1/\sqrt{1-\beta^2}$,

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{4^2}{5^2}} = \sqrt{\frac{9}{25}} = \frac{3}{5}.$$

That is, $v = (3/5)c$.

15.12 ⋆⋆ The half-life in the pions' rest frame is the proper half-life, $t_{1/2}(\text{proper}) = 1.8 \times 10^{-8}$ s. The length of the pipe as "seen" by the pions is given by the length-contraction formula

$$(\text{length of pipe in pions' frame}) = d/\gamma = 21.6 \text{ m},$$

where $d = 36$ m is the length measured in the lab. The time for the pipe to pass the pions is therefore $T(\pi \text{ frame}) = (d/\gamma)/v = d/(\gamma v)$, and the number of half-lives that elapse is

$$n = \frac{T(\pi \text{ frame})}{t_{1/2}(\pi \text{ frame})} = \frac{d}{\gamma v t_{1/2}(\text{proper})} = 5.00. \qquad \text{(vii)}$$

Therefore the number of pions that survive is $N = N_o/2^n = 32,000/32 = 1000$, the same answer as in Problem 15.8.

The two arguments, in this problem and Problem 15.8, lead to the same formula, (vii) here and (v) in the solution to Problem 15.8. Here the factor of γ comes from the length contraction of the tube, there the same factor came from the time dilation of the time of flight.