36. The easiest way to do this problem is to use the fact that $C = S/100$, so

$$\frac{dC}{dt} = \frac{1}{100}\frac{dS}{dt}.$$

Hence,

$$\frac{dC}{dt} = \frac{1}{100}\frac{dS}{dt} = \frac{1}{100}\left(20 - \frac{3S}{100}\right) = \frac{1}{100}(20 - 3C)$$

We obtain

$$\frac{dC}{dt} = \frac{20 - 3C}{100} = \frac{1}{5} - \frac{3C}{100}.$$

We can also derive this differential equation directly, but in order to do so, we need to determine how the concentrations of the incoming mixtures affect the total concentration in the vat. Since the total concentration $C(t) = S(t)/V(t)$ where $V(t)$ is the total volume,

$$\begin{aligned} \frac{dC}{dt} &= \frac{1}{V}\frac{dS}{dt} - \frac{1}{V^2}\frac{dV}{dt}S \\ &= \frac{1}{V}\frac{dS}{dt} - \frac{1}{V}\frac{S}{V}\frac{dV}{dt} \\ &= \frac{\dfrac{dS}{dt} - C\dfrac{dV}{dt}}{V}. \end{aligned}$$

If we want to consider only the contribution to dC/dt from one pipe, we let C_{new} be the concentration of the sugar entering from that pipe alone. Then

$$\frac{dS}{dt} = C_{new}\frac{dV}{dt},$$

where both derivatives represent the rates associated to the pipe in question. Considering just the one pipe, the formula for dC/dt is

$$\frac{dC}{dt} = \frac{(C_{new} - C)\dfrac{dV}{dt}}{V}.$$

(When using this formula, it is important to realize that both dC/dt and dV/dt refer only to the pipe in question, but V refers to the total volume.)

For Pipe A, we have $C_{new} = 5$ and $dV/dt = 2$. For Pipe B, we have $C_{new} = 10$ and $dV/dt = 1$. For Pipe C, we have $C_{new} = C$ and $dV/dt = -3$. In this case, $V(t) = 100$ for all t. Totaling the contributions to dC/dt for all three pipes gives us

$$\frac{dC}{dt} = \frac{(5 - C)2 + (10 - C)1 + (C - C)3}{100}.$$

We obtain

$$\frac{dC}{dt} = \frac{20 - 3C}{100}.$$

37. Let $C(t)$ denote the concentration. From Exercise 36, the differential equation is

$$\frac{dC}{dt} = \frac{20 - 3C}{100}$$

Note that $dC/dt = 0$ for all t if $C = 20/3$. Hence, the constant function $C(t) = 20/3$ is an equilibrium solution.

Separating variables and integrating, we obtain

$$\begin{aligned} \int \frac{dC}{20 - 3C} &= \int \frac{1}{100}\,dt \\ -\frac{1}{3}\ln|20 - 3C| &= \frac{t}{100} + c \\ |20 - 3C| &= c_1 e^{-3t/100}. \end{aligned}$$

To remove the absolute value signs we note that $e^{-3t/100}$ is never zero, so $20 - 3C(t)$ is either positive for all t, negative for all t, or zero for all t. Hence the general solution is

$$20 - 3C = k_1 e^{-3t/100},$$

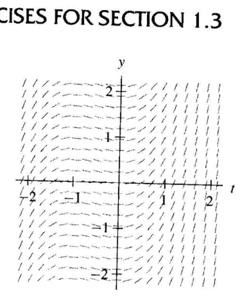
where k_1 can be any constant. Finally, we have

$$C(t) = \tfrac{20}{3} + ke^{-3t/100}$$

where k can be any constant.

EXERCISES FOR SECTION 1.3

1.



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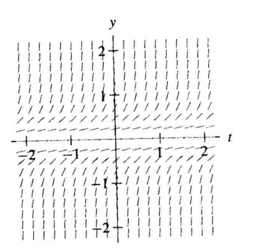
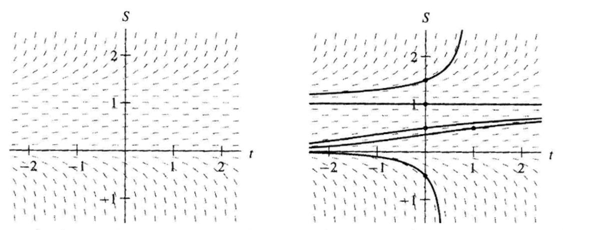


12.



15. **(a)** Note that the slopes are constant along vertical lines—lines along which t is constant, so the right-hand side of the corresponding equation depends only on t. The only choices are equations (i) and (iv). Because the slopes are negative for $t > 1$ and positive for $t < 1$, this slope field corresponds to equation (iv).

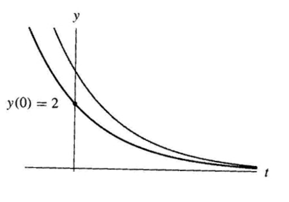
(b) This slope field has an equilibrium solution corresponding to the line $y = 1$, as does equations (ii), (v), (vii), and (viii). Equations (ii), (v), and (viii) are autonomous, and this slope field is not constant along horizontal lines. Consequently, it corresponds to equation (vii).

(c) This slope field is constant along horizontal lines, so it corresponds to an autonomous equation. The autonomous equations are (ii), (v), and (viii). This field does not correspond to equation (v) because it has the equilibrium solution $y = -1$. The slopes are negative between $y = -1$ and $y = 1$. Consequently, this field corresponds to equation (viii).

(d) This slope field depends both on y and on t, so it can only correspond to equations (iii), (vi), or (vii). It does not correspond to (vii) because it does not have an equilibrium solution at $y = 1$. Also, the slopes are positive if $y > 0$. Therefore, it must correspond to equation (vi).

17. **(a)** Because the equation is autonomous, the slope field is constant on horizontal lines, so this solution provides enough information to sketch the slope field on the entire upper half plane. Also, if we assume that f is continuous, then the slope field on the line $y = 0$ must be horizontal.

(b) The solution with initial condition $y(0) = 2$ is a translate to the left of the given solution.



14.

Table 1.12
Results of Euler's method with $\Delta t = 1.0$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	1	2	1.41
2	2	3.41	1.85
3	3	5.26	2.29
4	4	7.56	

Table 1.13
Results of Euler's method with $\Delta t = 0.5$ (shown to two decimal places)

k	t_k	y_k	m_k
0	0	1	1
1	0.5	1.5	1.22
2	1.0	2.11	1.45
3	1.5	2.84	1.68
4	2.0	3.68	1.92
5	2.5	4.64	2.15
6	3.0	5.72	2.39
7	3.5	6.91	2.63
8	4.0	8.23	

Table 1.14
Results of Euler's method with $\Delta t = 0.25$ (shown to two decimal places)

k	t_k	y_k	m_k	k	t_k	y_k	m_k
0	0	1	1	9	2.25	4.32	2.08
1	0.25	1.25	1.12	10	2.50	4.84	2.20
2	0.50	1.53	1.24	11	2.75	5.39	2.32
3	0.75	1.84	1.36	12	3.0	5.97	2.44
4	1.0	2.18	1.48	13	3.25	6.58	2.56
5	1.25	2.55	1.60	14	3.50	7.23	2.69
6	1.50	2.94	1.72	15	3.75	7.90	2.81
7	1.75	3.37	1.84	16	4.0	8.60	
8	2.0	3.83	1.96				

The slopes in the slope field are positive and increasing. Hence, the graphs of all solutions are concave up. Since Euler's method uses line segments to approximate the graph of the actual solution, the approximate solutions will always be less than the actual solution. This error decreases as the step size decreases.