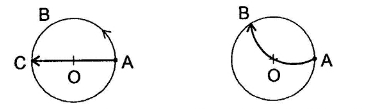
1.27 ⋆⋆ Since the puck is frictionless, the net force on it is zero, and, as seen from the ground, it travels in a straight line through the center O, as shown in the left picture. It starts from the point A at $t = 0$, travels "due west" with constant speed v_o, and falls onto the ground at point C after a time $T = 2R/v_o$ (where R is the radius of the turntable).

Now imagine an observer sitting on the turntable near A. As seen from the ground, he is traveling north with speed ωR. Therefore, as seen by the observer, the puck's initial velocity has a sideways (southerly) component ωR, in addition to the westerly component v_o; that is, the puck moves initially west *and* south, as shown in the right picture. (The magnitude of the southerly component depends on the table's rate of rotation ω.) As the puck moves in to a smaller radius r, the sideways component ωr gets less, so the puck's path curves to the right. Continuing to curve, its passes through O and eventually reaches the edge of the turntable at point B. The left picture shows the point B of the table at time $t = 0$. The position of B is determined by the following consideration: In the time $T = 2R/v_o$ for the puck to cross the table, point B of the table must move around to point C where we know the puck falls to the ground. Thus the angle BOC is equal to ωT. The faster the table rotates, the larger the angle BOC and the more sharply the puck's path (as seen from the table) is curved.



seen from ground   seen from tumtable

1.45 ⋆⋆ Since the magnitude of $\mathbf{v}(t)$ is the same as $\sqrt{\mathbf{v}(t)\cdot\mathbf{v}(t)}$, the magnitude is constant if and only if $\mathbf{v}(t)\cdot\mathbf{v}(t)$ is. Since

$$\frac{d}{dt}[\mathbf{v}(t)\cdot\mathbf{v}(t)] = 2\mathbf{v}(t)\cdot\dot{\mathbf{v}}(t),$$

this implies that the magnitude of $\mathbf{v}(t)$ is constant if and only if $\mathbf{v}(t)\cdot\dot{\mathbf{v}}(t) = 0$; that is, $\mathbf{v}(t)$ is orthogonal to $\dot{\mathbf{v}}(t)$

1.46 ⋆⋆ **(a)** As seen in the inertial frame $\mathcal{S}$ the puck moves in a straight line with $\phi = 0$ and $r = R - v_o t$

(b) As seen in $\mathcal{S}'$, $r' = r = R - v_o t$ and $\phi' = \phi - \omega t = -\omega t$. This path is sketched in the answer to Problem 1.27. Initially, the puck moves inward with speed v_o but also downward with speed ωR. It curves to its right, passing through the center and continuing to curve to the right until it slides off the turntable.

2.2 ⋆ According to Stokes's law $f_{\text{lin}} = (3\pi\eta D)v$, which has precisely the form $f_{\text{lin}} = bv$ if we define $b = \beta D$ and $\beta = 3\pi\eta = 3\pi(1.7 \times 10^{-5}\ \text{N}\cdot\text{s/m}^2) = 1.6 \times 10^{-4}\ \text{N}\cdot\text{s/m}^2$.

2.3 ⋆ **(a)** From (2.84) and (2.82), $f_{\text{quad}}/f_{\text{lin}} = (\kappa\varrho A v^2)/(3\pi\eta D v)$. With $\kappa = 1/4$ and $A = \pi D^2/4$, this becomes $\varrho D v/(48\eta)$ or $R/48$, with R given by (2.83).

(b) With the given numbers, $R = 1.1 \times 10^{-2}$ and it is very safe to neglect the quadratic drag.

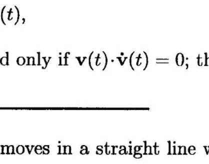
2.4 ⋆⋆ **(a)** In a short time dt the projectile moves a distance vdt, and the front sweeps out a cylinder of volume $Avdt$. Therefore the mass of fluid encountered is $\varrho Avdt$, and the rate at which mass is swept up is ϱAv.

(b) If a mass $\varrho Avdt$ is accelerated from 0 to v in time dt, the rate of change of its momentum is ϱAv^2. This is, therefore, the forward force on the fluid and, hence, the backward force on the projectile.

(c) Since $A \propto D^2$, it follows that $f_{\text{quad}} = \kappa\varrho Av^2 = cv^2$, where $c = \kappa\varrho A \propto D^2$. For a sphere in air, $\kappa = 1/4$, $A = \pi D^2/4$, and $\varrho = 1.29\ \text{kg/m}^3$, so $f_{\text{quad}} = (\kappa\varrho\pi D^2/4)v^2 = cv^2$, where $c = \gamma D^2$ and

$$\gamma = \kappa\varrho\pi/4 = \tfrac{1}{4} \times (1.29\ \text{kg/m}^3) \times \pi/4 = 0.25\ \text{N}\cdot\text{s}^2/\text{m}^4.$$

2.5 ⋆ With $v_y > v_{\text{ter}}$, the drag force is greater than the weight, and the net force is upward. Thus the projectile slows down, with v_y approaching v_{ter} as $t \to \infty$. This is clear from Eq.(2.30), as shown in the plot.



2.6 ⋆ **(a)** If we insert the Taylor series for $e^{-t/\tau}$ into (2.33), we get

$$v_y(t) = v_{\text{ter}}\left[1 - e^{-t/\tau}\right] = v_{\text{ter}}\left[1 - \left(1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2} - \cdots\right)\right].$$

The first two terms on the right cancel, and, if t is sufficiently small, we can neglect terms in t^2 and higher. This leaves us with

$$v_y(t) \approx v_{\text{ter}}t/\tau = gt$$

where to get the second equality I replaced v_{ter} by $g\tau$ as in (2.34).

(b) Putting $v_{yo} = 0$ into (2.35) and then inserting the Taylor series for the exponential, we find:

$$y(t) = v_{\text{ter}}t - v_{\text{ter}}\tau\left[1 - e^{-t/\tau}\right] = v_{\text{ter}}t - v_{\text{ter}}\tau\left[1 - \left(1 - \frac{t}{\tau} + \frac{t^2}{2\tau^2} - \cdots\right)\right].$$

On the right side, the second and third terms cancel, as do the first and fourth. If we neglect all terms beyond t^2, this leaves us with $y(t) \approx v_{\text{ter}}t^2/(2\tau) = \frac{1}{2}gt^2$, since $v_{\text{ter}} = g\tau$.

2.8 ⋆ $$t = m\int_{v_o}^{v} \frac{dv'}{-cv'^{3/2}} = \frac{2m}{c}\left[v'^{-1/2}\right]_{v_o}^{v} = \frac{2m}{c}\left(\frac{1}{\sqrt{v}} - \frac{1}{\sqrt{v_o}}\right)$$

or, solving for v, $v = v_o/(1 + ct\sqrt{v_o}/2m)^2$. Clearly, $v = 0$ only when $t \to \infty$.

2.11 ⋆⋆ **(a)** Since we are now measuring y upward, the answers can be found from (2.30) and (2.35) by replacing v_{ter} with $-v_{\text{ter}}$:

$$v_y(t) = -v_{\text{ter}} + (v_o + v_{\text{ter}})e^{-t/\tau} \quad \text{and} \quad y(t) = -v_{\text{ter}}t + (v_o + v_{\text{ter}})\tau(1 - e^{-t/\tau}).$$

(b) Setting $v_y = 0$ and solving for t, we find $t_{\text{top}} = \tau\ln(1 + v_o/v_{\text{ter}})$. Sustituting this time into $y(t)$ we find $y_{\text{max}} = [v_o - v_{\text{ter}}\ln(1 + v_o/v_{\text{ter}})]\tau$.

(c) In the vacuum $v_{\text{ter}} = \infty$. Letting $v_{\text{ter}} \to \infty$ in y_{max} and using the suggested approximation for the log term, we find

$$y_{\text{max}} \to \left\{v_o - v_{\text{ter}}\left[\frac{v_o}{v_{\text{ter}}} - \frac{1}{2}\left(\frac{v_o}{v_{\text{ter}}}\right)^2\right]\right\}\tau = \frac{v_o^2}{2g}$$

since the first two terms in the middle expression cancel each other and $v_{\text{ter}} = g\tau$.

2.16 ⋆ As usual, $x = (v_o\cos\theta)t$ and $y = (v_o\sin\theta)t - \frac{1}{2}gt^2$. The time to reach the plane of the wall ($x = d$) is $t = d/(v_o\cos\theta)$ and the ball's height at that time is $y = d\tan\theta - \frac{1}{2}gd^2/(v_o\cos\theta)^2$. Notice that this height decreases monotonically as v_o decreases. Thus there is indeed a minimum speed $v_o(\text{min})$ for which the ball clears the wall. Putting $y = h$ and solving for v_o we find that

$$v_o(\text{min}) = \sqrt{\frac{gd^2}{2(d\tan\theta - h)\cos^2\theta}}.$$

If $\tan\theta < h/d$, the argument of the square root is negative and there is no real $v_o(\text{min})$; physically, the ball's initial velocity is aimed below the top of the wall, so the ball cannot possibly clear the wall whatever its speed. With the given numbers, $v_o(\text{min}) = 26.4$ m/s or roughly 50 mi/hr.