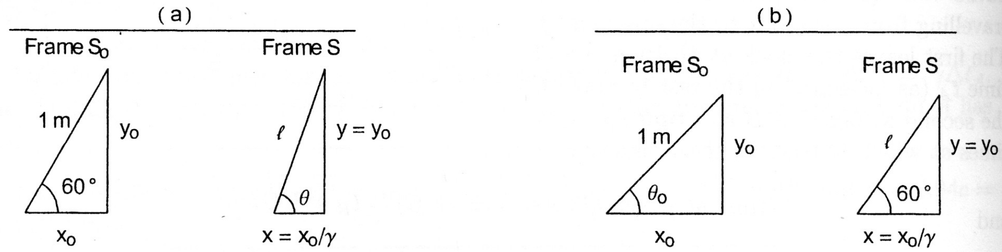
Taylor Solutions Relativity Week 2

15.13 $\star\star$



(a) With $\beta = 4/5$, $\gamma = 5/3$. In the frame $\mathcal{S}_o$, we know the length $l_o = 100$ cm and the angle $\theta_o = 60°$, so we can calculate $x_o = 50$ cm and $y_o = 86.6$ cm. In the frame $\mathcal{S}$, x is contracted ($x = x_o/\gamma = 30$ cm) but y is not ($y = y_o = 86.6$ cm). Thence $l = \sqrt{x^2 + y^2} = 91.7$ cm and $\theta = \arctan(y/x) = 70.9°$.

(b) The angle 60° is given in the frame $\mathcal{S}$, so $\tan 60° = y/x = y_o/(x_o/\gamma)$ and $\tan\theta_o = y_o/x_o = (\tan 60°)/\gamma$, whence $\theta_o = 46.1°$. From this we find $x_o = 69.3$ cm and $y_o = 72.1$ cm, and from these we can calculate $x = x_o/\gamma$ and $y = y_o$ and thence $l = 83.2$ cm.

15.18 $\star\star$ The inverse Lorentz transformation (15.21) applied to the flash and beep of Figure 15.3 implies that

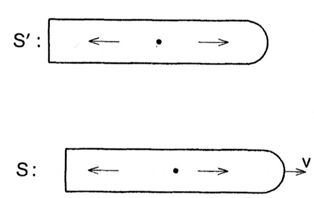
$$t_{\text{beep}} = \gamma(t'_{\text{beep}} + \beta x'_{\text{beep}}/c) \quad \text{and} \quad t_{\text{flash}} = \gamma(t'_{\text{flash}} + \beta x'_{\text{flash}}/c)$$

Now, in frame $\mathcal{S}'$ the flash and beep occur at the same place, so $x'_{\text{beep}} = x'_{\text{flash}}$. Therefore, if we subtract the first of these equations from the second, we find

$$\Delta t = \gamma \Delta t'$$

which is the time-dilation formula.

15.19 $\star\star$ **(a)** $x'_F = d$, $t'_F = d/c$; $x'_B = -d$, $t'_B = d/c$.



(b)

$$x_F = \gamma(x'_F + vt'_F) = \gamma(1+\beta)d$$
$$t_F = \gamma(t'_F + vx'_F/c^2) = \gamma(1+\beta)d/c$$
$$x_B = \gamma(x'_B + vt'_B) = -\gamma(1-\beta)d$$
$$t_B = \gamma(t'_B + vx'_B/c^2) = \gamma(1-\beta)d/c$$

Although the two events are simultaneous as measured in S', they are *not* simultaneous in S. As observed in S, the two signals start out from the middle of the rocket, but while they are traveling the rocket is also traveling to the right at speed v. Thus the front is receding from its signal, which must travel more than half the rocket's length. Meanwhile the back of the rocket is approaching its signal, which needs to travel only a shorter distance. Therefore this signal arrives first; that is, $t_B < t_F$. (In S' the signals again start from the middle of the rocket; but since the rocket is not moving they naturally arrive simultaneously.)

15.21 ⋆ Let us take our x axis in the direction of the two velocities. Then the velocity of the rocket's frame $\mathcal{S}'$ has $V = \frac{1}{2}c$ and that of the bullets relative to the rocket has $v'_x = \frac{3}{4}c$, with all other components zero. According to the inverse of the velocity-addition formula (15.26),

$$v_x = \frac{v'_x + V}{1 + v'_x V/c^2} = \frac{\frac{1}{2} + \frac{3}{4}}{1 + \frac{3}{8}} c = \frac{5/4}{11/8} c = \frac{10}{11} c,$$

with all other components zero.

15.24 ⋆ Let $\mathcal{S}$ be the frame fixed to the ground and $\mathcal{S}'$ the one fixed to the cop's car. The velocity of $\mathcal{S}'$ relative to $\mathcal{S}$ is $V = 0.4c$ and the velocity of the bullet relative to $\mathcal{S}'$ is $v = 0.5c$. By the inverse velocity transformation, the bullet's velocity relative to the ground is

$$v = \frac{v' + V}{1 + v'V/c^2} = \frac{0.4 + 0.5}{1 + 0.4 \times 0.5} c = 0.75c.$$

Since the robber's velocity relative to the ground is $0.8c$, the bullets do not catch the robber.

15.26 ⋆ The two positions are $x_A = v_A t$ and $x_B = d - v_B t$. The two objects meet when $x_A = x_B$ or $v_A t = d - v_B t$; that is, $t = d/(v_A + v_B)$.
