[redacted]

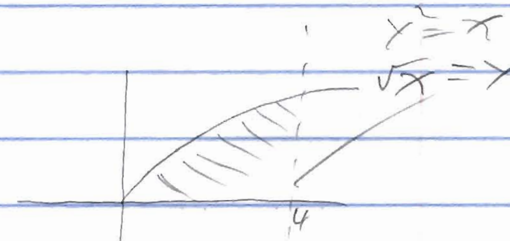
5.2.27

$$\int_0^4 \int_0^{\sqrt{x}} y\sqrt{x}\, dy\, dx$$

$$\frac{1}{2}\int_0^4 y^2\Big]_0^{\sqrt{x}} \sqrt{x}\, dx$$

$$\frac{1}{2}\int_0^4 x^{3/2}\, dx$$

$$\frac{1}{5} x^{5/2}\Big]_0^4 = \frac{32}{5}$$



$y^2 \le x \le 4$

$0 \le y \le 2$

$$\int_0^2 \int_{y^2}^4 y\sqrt{x}\, dx\, dy$$

$$\frac{2}{3}\int_0^2 y \; x^{3/2}\Big]_{y^2}^4 dy$$

$$\frac{2}{3}\int_0^2 y(8 - y^3)\, dy$$

$$\int_0^2 8y - y^4\, dy$$

$$\frac{2}{3}\left[4y^2 - \frac{y^5}{5}\right]_0^2$$

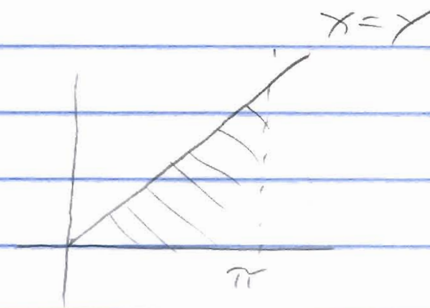
$$\frac{2}{3}\left[16 - \frac{32}{5}\right]$$

$$\frac{32}{3} - \frac{64}{15}$$

$$\frac{160}{15} - \frac{64}{15} = \frac{96}{15} = \frac{32}{5}$$

5.2.29

$$\int_0^\pi \int_y^\pi \frac{\sin x}{x}\,dx\,dy$$



$0 \le y \le x$

$0 \le x \le \pi$

$$\int_0^\pi \int_0^x \frac{\sin x}{x}\,dy\,dx$$

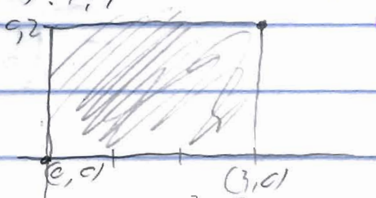
$$\int_0^\pi \frac{\sin x}{x}\,y\Big]_0^x\,dx$$

$$\int_0^\pi \sin x\,dx$$

$$-\cos x\Big]_0^\pi$$

$1 - -1 = 2$

5.3.7



$\rho = xy$

a) $M = \int_0^3 \int_0^2 xy \, dy \, dx$

$\int_0^3 \left[x \frac{y^2}{2} \right]_0^2 dx$

$2 \int_0^3 x \, dx$

$x^2 \Big]_0^3 = 9$

b) $\bar{x} \int_0^3 \int_0^2 xy \, dy \, dx = \int_0^3 \int_0^2 x^2 y \, dy \, dy$

$9\bar{x} = 9 \cdot 2$

$\bar{x} = 2$

$9\bar{y} = \int_0^3 \int_0^2 x y^2 \, dy \, dx$

$\bar{y} = \frac{8}{3} \cdot \frac{9}{2} \cdot \frac{1}{9}$

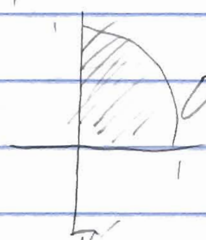
$\bar{y} = 4/3$

center of mass $= (2, 4/3)$

c) $I_x = \int_0^2 \int_0^3 y^2 xy \, dx \, dy = \left[\frac{x^2}{2} \right]_0^3 \cdot \left[\frac{y^4}{4} \right]_0^2 = \frac{9}{2} \cdot 4 = 2M$

$I_y = \int_0^2 y \, dy \cdot \int_0^3 x^3 \, dx = 2 \left[\frac{x^4}{4} \right]_0^3 = \frac{81}{2} = \frac{9M}{2}$

5.4.14



$0 \le r \le 1 \quad 0 \le \theta \le \frac{\pi}{4}$

$$\int_0^{\pi/2} d\theta \int_0^1 e^{-r^2}\, r\, dr$$

$$\frac{\pi}{2} \cdot \frac{-1}{2}\left[e^{-r^2}\right]_0^1$$

$$-\frac{\pi}{4}\left[e^{-1}-1\right]$$