6. Note that $dy/dt = 0$ if $y = 3$. Hence, $y_1(t) = 3$ for all t is an equilibrium solution. By the Uniqueness Theorem, this is the only solution that is 3 at $t = 0$. Therefore, $y(t) = 3$ for all t.

7. Because $0 < y(0) < 2$ and $y_1(t) = 0$ and $y_2(t) = 2$ are equilibrium solutions of the differential equation, we know that $0 < y(t) < 2$ for all t by the Uniqueness Theorem. Also, $dy/dt > 0$ for $0 < y < 2$, so dy/dt is always positive for this solution. Hence, $y(t) \to 2$ as $t \to \infty$, and $y(t) \to 0$ as $t \to -\infty$.

17. **(a)** The equation is separable. Separating variables we obtain

$$\int (y-2)\,dy = \int t\,dt.$$

Solving for y with help from the quadratic formula yields the general solution

$$y(t) = 2 \pm \sqrt{t^2 + c}\,.$$

To find c, we let $t = -1$ and $y = 0$, and we obtain $c = 3$. The desired solution is therefore $y(t) = 2 - \sqrt{t^2 + 3}$

(b) Since $t^2 + 2$ is always positive and $y(t) < 2$ for all t, the solution $y(t)$ is defined for all real numbers.

(c) As $t \to \pm\infty$, $t^2 + 3 \to \infty$. Therefore,

$$\lim_{t\to\pm\infty} y(t) = -\infty.$$

18. **(a)** The partial derivative with respect to v of dv/dt does not exist at $v = 0$. Hence the Uniqueness Theorem tells us nothing about the uniqueness of solutions that involve $v = 0$. In fact, if we use the techniques described in the section related to the uniqueness of solutions for $dy/dt = 3y^{2/3}$, we can find infinitely many solutions with this initial condition.

(b) Since it does not make sense to talk about rain drops with negative volume, we always have $v \geq 0$. Once $v > 0$, the evolution of the drop is completely determined by the differential equation.

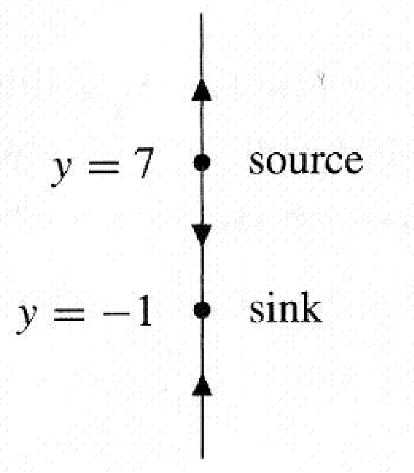
What is the physical significance of a drop with $v = 0$? It is tempting to interpret the fact that solutions can have $v = 0$ for an arbitrary amount of time before beginning to grow as a statement that the rain drops can spontaneously begin to grow at any time. Since the model gives no information about when a solution with $v = 0$ starts to grow, it is not very useful for the understanding the initial formation of rain drops. The safest assertion is to say is the model breaks down if $v = 0$.

1.6 Equilibria and the Phase Line

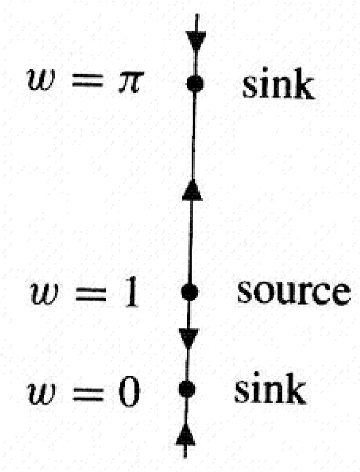
2. The equilibrium points of $dy/dt = f(y)$ are the numbers y where $f(y) = 0$. For

$$f(y) = y^2 - 6y - 7 = (y-7)(y+1),$$

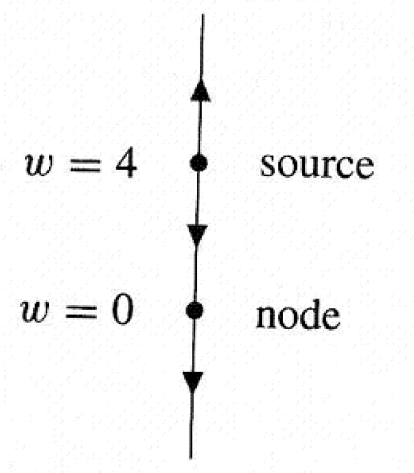
the equilibrium points are $y = -1$ and $y = 7$. Since $f(y)$ is positive for $y < -1$, negative for $-1 < y < 7$, and positive for $y > 7$, the equilibrium point $y = -1$ is a sink and the equilibrium point $y = 7$ is a source.



5. The equilibrium points of $dw/dt = f(w)$ are the numbers w where $f(w) = 0$. For $f(w) = (w-1)\sin w$, the equilibrium points are $w = 1$ and $w = n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$. The sign of $(w-1)\sin w$ alternates between positive and negative at successive zeros. It is positive for $-\pi < w < 0$ and negative for $0 < w < 1$. Therefore, $w = 0$ is a sink, and the equilibrium points alternate between sources and sinks.



8. The equilibrium points of $dw/dt = f(w)$ are the numbers w where $f(w) = 0$. For $f(w) = 3w^3 - 12w^2$, the equilibrium points are $w = 0$ and $w = 4$. Since $f(w) < 0$ for $w < 0$ and $0 < w < 4$, and $f(w) > 0$ for $w > 4$, the equilibrium point at $w = 0$ is a node and the equilibrium point at $w = 4$ is a source.



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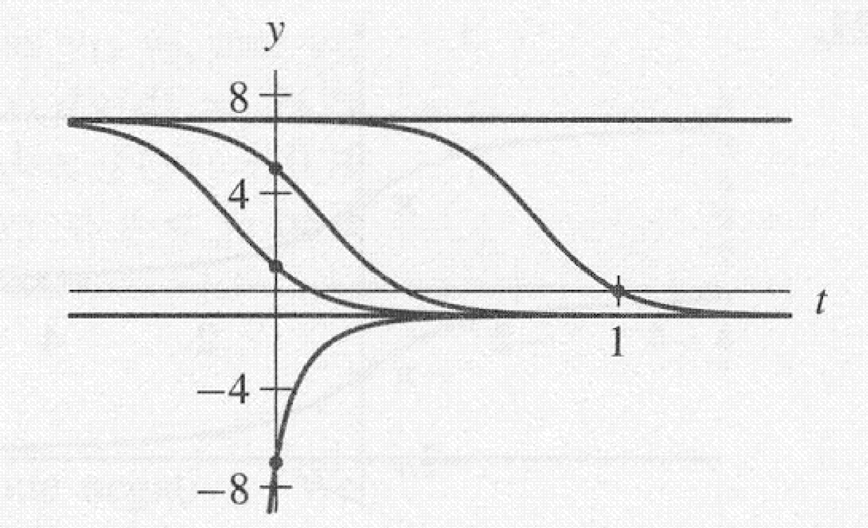

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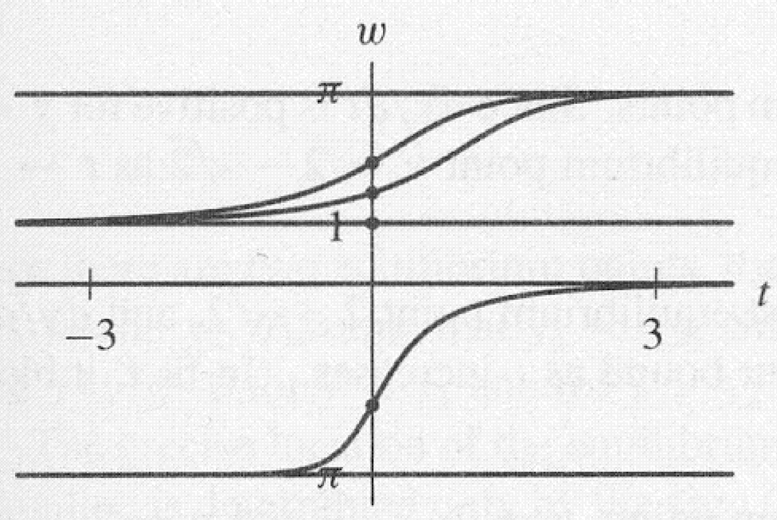

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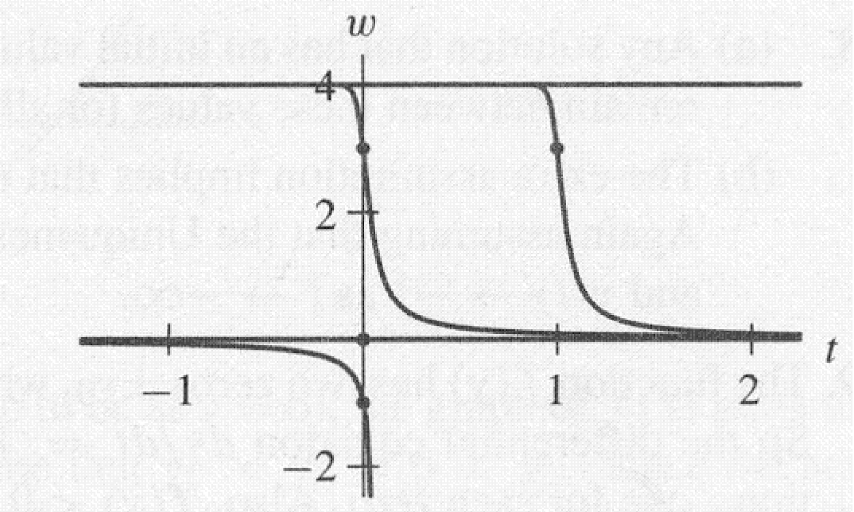
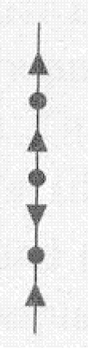


30. The function $f(y)$ has three zeros. We denote them as y_1, y_2, and y_3, where $y_1 < 0 < y_2 < y_3$. So the differential equation $dy/dt = f(y)$ has three equilibrium solutions, one for each zero. Also, $f(y) > 0$ if $y < y_1$, $f(y) < 0$ if $y_1 < y < y_2$, and $f(y) > 0$ if $y_2 < y < y_3$ or if $y > y_3$. Hence y_1 is a sink, y_2 is a source, and y_3 is a node.



37. **(a)** This phase line has three equilibrium points, $y = 0, \pm 1$. Only equations (vii) and (viii) have three equilibria. For this phase line, $dy/dt < 0$ for $0 < y < 1$. Only equation (vii) satisfies this property. Consequently, the phase line corresponds to equation (vii).

(b) This phase line has two equilibrium points, $y = 0$ and $y = 1$. Equations (i), (ii), (iii), and (iv) have exactly these equilibria. For this phase line, $dy/dt \geq 0$ for $y > 0$. Only equations (i) and (ii) satisfy this property. Moreover, for this phase line, $dy/dt < 0$ for $y < 0$. Only equation (ii) satisfies this property. Consequently, the phase line corresponds to equation (ii).

(c) This phase line has two equilibrium points, $y = 0$ and $y = 2$. Equations (v) and (vi) have exactly these equilibria. For this phase line, $dy/dt > 0$ for $0 < y < 2$. Only equation (vi) satisfies this property. Consequently, the phase line corresponds to equation (vi).

(d) This phase line has two equilibrium points, $y = 0$ and $y = 1$. Equations (i), (ii), (iii), and (iv) have exactly these equilibria. For this phase line, $dy/dt < 0$ for $0 < y < 1$. Only equation (iii) satisfies this property. Consequently, the phase line corresponds to equation (iii).