Worksheet

1) $\vec{F}(t) = \langle \sin(t), \cos(t), 1 \rangle$

a) $\vec{F}'(t) = \langle \cos t, -\sin t, 0 \rangle$

b) $\frac{dz}{dt} = 0 \implies \vec{F}'(t) \parallel xy\text{-plane}$

c) $-\sin t = 0$

$\sin t = 0$

$t = \sin^{-1}(0)$

$t = n\pi \quad n \in \mathbb{Z}$

d) $|\vec{F}(t)| = \sqrt{\sin^2 t + \cos^2 t + 1^2} = \sqrt{1+1} = \sqrt{2}$ yes

e) $|\vec{F}'(t)| = \sqrt{\cos^2 t + (-\sin t)^2} = \sqrt{1} = 1$

f) $\vec{F}''(t) = \langle -\sin t, -\cos t, 0 \rangle$

8) a) Determine arclength of $\vec{R}(t) = \langle e^t \cos t, e^t \sin t, 0 \rangle$ on the interval $0 \le t \le 1$.

$$s = s(t) = \int \left|\frac{dR}{dt}\right| dt$$

$$\frac{dR}{dt} = \langle e^t \cos t - e^t \sin t, e^t \sin t + e^t \cos t, 0 \rangle$$

$$\left|\frac{dR}{dt}\right| = \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \cos t + e^t \sin t) + 0}$$

$$= \sqrt{2e^{2t}\cos^2 t + 2e^{2t}\sin^2 t}$$

$$= \sqrt{2e^{2t}}$$

$$= \sqrt{2}\, e^t$$

$$s = \int_0^1 \sqrt{2} e^t \, dt = \sqrt{2}\left[e^t\right]_0^1 = \sqrt{2}e - \sqrt{2}$$

b) $s(t) = \sqrt{2}\int_0^t e^t \, dt = \sqrt{2}(e^t - 1)$

$s = \sqrt{2}(e^t - 1)$

$\frac{s}{\sqrt{2}} + 1 = e^t$

$\ln\left|\frac{s}{\sqrt{2}} + 1\right| = t$

so

$$\vec{R}(s) = \left\langle \left(\frac{s}{\sqrt{2}} + 1\right)\cos\left(\ln\left|\frac{s}{2} + 1\right|\right), \left(\frac{s}{\sqrt{2}} + 1\right)\sin\left(\left|\ln\left|\frac{s}{\sqrt{2}} + 1\right|\right), 0 \right\rangle$$

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5) $x^2 \ln(1-x) = x^2\left(-x - \frac{x^2}{2} - \frac{x^3}{3} - \dots\right.$

$= -x^3 - \frac{x^4}{2} - \frac{x^5}{3} - \dots$

$= -\sum_{n=1}^{\infty} \frac{x^{n+2}}{n}$

20) $x + x^2 + \frac{x^3}{3} - \frac{x^5}{30} - \frac{x^6}{90} \dots$