17

6.8.17

$F_1 = \langle -y, x, z \rangle$ $\qquad$ $F_2 = \langle y, x, z \rangle$

$\nabla \times F_1 = \langle 0, 0, 2 \rangle$ $\qquad$ $\nabla \times F_2 = \langle 0, 0, 0 \rangle$

$\therefore$ F_2 is conservative & F_1 nonconservative.

Since F_2 is conservative $W = 0$.

For F_1 we have

$x = \cos t \implies dx = -\sin t\, dt$

$y = \sin t \implies dy = \cos t\, dt$

$z = 0 \implies dz = 0$

So

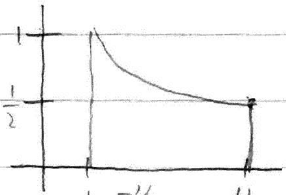
$$W = \int F \cdot dr = \int_0^{2\pi} (-\sin t)(-\sin t)\,dt + \cos t \cos t\, dt + 0$$

$$= \int_0^{2\pi} (\sin^2 t + \cos^2 t)\,dt$$

$$= t \Big]_0^{2\pi} = 2\pi$$

6.9.3

$$\oint_C xy\,dx + x^2\,dy$$



$$0 \le y \le \frac{1}{\sqrt{x}}$$
$$1 \le x \le 4$$

using Green's Thm

$$\oint_C xy\,dx + x^2\,dy = \int_1^4 \int_0^{x^{-1/2}} 2x - x \,dy\,dx$$

$$= \int_1^4 \int_0^{x^{-1/2}} x\,dy\,dx$$

$$= \int_1^4 x^{1/2}\,dx$$

$$= \frac{2}{3}\left[x^{3/2}\right]_1^4$$

$$= \frac{2}{3}(8-1) = \frac{14}{3}$$

6.9.6

$$\frac{1}{2}\oint_C -y\,dx + x\,dy = \frac{1}{2}\iint_A 1+1\,dA$$

$$= \frac{1}{2}\iint_A 2\,dA$$

$$= \iint_A dA = A = \frac{1}{2}\oint_C -y\,dx + x\,dy$$

6.9.7

$$A = \frac{1}{2} \oint_C x\,dy - y\,dx$$

let $x = a\cos\theta \Rightarrow dx = -a\sin\theta\,d\theta$ $\quad 0 \le \theta \le 2\pi$

$y = b\sin\theta \Rightarrow dy = b\cos\theta\,d\theta$

$$A = \frac{1}{2} \int_0^{2\pi} ab\sin^2\theta\,d\theta + ab\cos^2\theta\,d\theta$$

$$= \frac{ab}{2} \int_0^{2\pi} d\theta$$

$$= \pi ab$$