6.7.8

Compute div & curl of $\langle \sinh z, 2y, x\cosh z \rangle = \vec{f}(x,y,z)$

$\nabla \cdot f = 0 + 2 + x\sinh z = 2 + x\sinh z$

$\nabla \times f = i(0-0) - j(\cosh z - \cosh z) + k(0-0) = \vec{0}$

6.7.14

$f = (x^2+y^2+z^2)^{-1/2}$

find $\nabla^2 \cdot f$

$$\nabla f = \langle -x(x^2+y^2+z^2)^{-3/2}, -y(x^2+y^2+z^2)^{-3/2}, -z(x^2+y^2+z^2)^{-3/2}$$

$$\begin{aligned}\nabla^2 f &= -(x^2+y^2+z^2)^{-3/2} + 3x^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + \\ &\quad 3y^2(x^2+y^2+z^2)^{-5/2} - (x^2+y^2+z^2)^{-3/2} + 3z^2(x^2+y^2+z^2)^{-5/2} \\ &= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)(x^2+y^2+z^2)^{-5/2} \\ &= -3(x^2+y^2+z^2)^{-3/2} + 3(x^2+y^2+z^2)^{-3/2} \\ &= 0\end{aligned}$$

6.7.17.b

$$\begin{bmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{bmatrix} = i\left(\frac{\partial^2 \phi}{\partial y \partial z} - \frac{\partial^2 \phi}{\partial z \partial y}\right) - j\left(\frac{\partial^2 \phi}{\partial z \partial x} - \frac{\partial^2 \phi}{\partial x \partial z}\right) + k\left(\frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial y \partial x}\right) = \vec{0}$$

Hey! the curl of a conservative field is 0

6.7.17.9

WTS: $\nabla \times (\phi \vec{V}) = \phi(\nabla \times V) - V \times (\nabla \phi)$

$$= \phi \left\langle \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}, \frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}, \frac{\partial V_x}{\partial y} - \frac{\partial V_x}{\partial z} \right\rangle -$$

$$\left\langle V_y \frac{\partial \phi}{\partial z} - V_z \frac{\partial \phi}{\partial y}, V_z \frac{\partial \phi}{\partial x} - V_x \frac{\partial \phi}{\partial z}, V_x \frac{\partial \phi}{\partial z} - V_z \frac{\partial \phi}{\partial x} \right\rangle$$

$$= \left\langle \phi\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - V_y \frac{\partial \phi}{\partial z} + V_z \frac{\partial \phi}{\partial y}, \right.$$

$$\phi\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) - V_z \frac{\partial \phi}{\partial x} + V_x \frac{\partial \phi}{\partial z},$$

$$\left. \phi\left(\frac{\partial V_x}{\partial y} - \frac{\partial V_y}{\partial x}\right) - V_x \frac{\partial \phi}{\partial x} + V_z \frac{\partial \phi}{\partial x} \right\rangle$$

$$= \left\langle \left(\phi \frac{\partial V_z}{\partial y} + V_z \frac{\partial \phi}{\partial y}\right) - \left(\phi \frac{\partial V_y}{\partial z} + V_y \frac{\partial \phi}{\partial z}\right), \right.$$

$$-\left(\phi \frac{\partial V_z}{\partial x} + V_z \frac{\partial \phi}{\partial x}\right) + \left(\phi \frac{\partial V_x}{\partial z} + V_x \frac{\partial \phi}{\partial z}\right),$$

$$\left. \left(\phi \frac{\partial V_y}{\partial x} + V_y \frac{\partial \phi}{\partial x}\right) - \left(\phi \frac{\partial V_x}{\partial y} + V_x \frac{\partial \phi}{\partial x}\right) \right\rangle$$

$$= \left\langle \frac{\partial \phi V_z}{\partial y} - \frac{\partial \phi V_y}{\partial z}, -\left(\frac{\partial \phi V_x}{\partial z} - \frac{\partial \phi V_z}{\partial x}\right), \frac{\partial \phi V_y}{\partial x} - \frac{\partial \phi V_x}{\partial y} \right\rangle$$

$$= \nabla \times (\phi \vec{V})$$